A Teacher's Guide to PISA Mathematics and Problem Solving

Findings from PISA 2012

Rachel Perkins and Gerry Shiel

Educational Research Centre

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Table of Contents

Preface

The Programme for International Assessment (PISA) is an international assessment of the skills and knowledge of 15-year-olds in mathematics, reading and science which is organised by the Organisation for Economic Cooperation and Development (OECD). PISA runs in three-yearly cycles, with one subject area becoming the main focus of the assessment in each cycle. In 2012, the fifth cycle of PISA, mathematics was the main focus of the assessment and reading and science were emphasised to a lesser extent. This is the second time since 2003 that mathematics was assessed as a major domain, allowing a detailed examination of changes in performance and attitudes towards mathematics over time. As part of PISA 2012, students in Ireland also participated in computer-based assessments of mathematics and reading (along with students in 31 other countries) and a computer-based assessment of problem solving (along with students in 43 other countries).

This guide is intended for teachers of mathematics in post-primary schools. It focuses on the performance of 15-year-old students in Ireland on both the print and computer-based assessments of mathematics, as well as the computer-based assessment of problem solving. All 23 Initial Project Maths schools were included in the sample for PISA in Ireland in 2012 and comparisons are drawn between the performance and attitudes of students in these schools and those in all other schools. This guide also examines student and school factors associated with achievement, including students' engagement with the PISA assessment.

This guide is divided into eight chapters. Chapter 1 provides an overview of the study, how it was conducted in Ireland and the relevance and role of PISA. Chapter 2 explains how PISA assesses mathematics and problem solving. Chapter 3 presents some examples of PISA questions from both the print and computer-based assessments. The performance of students in Ireland on print and computer-based mathematics, and on computer-based problem solving, is described in Chapter 4, while Chapter 5 examines student and school factors that are associated with achievement. Chapter 6 compares the performance and attitudes of students in the 23 Initial Project Maths schools to those in all other schools and Chapter 7 examines the relationship between students' engagement with the PISA assessments and performance on the test. Chapter 8 reflects on how the findings from PISA are relevant to the teaching and learning of mathematics in classrooms in Ireland.

A companion volume to this report, *PISA in Classrooms*, will be published in autumn 2016. That report will summarise the outcomes for mathematics from PISA 2012 and their implications for teaching and learning.

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Further Information on PISA

OECD PISA website: http://www.oecd.org/pisa

(includes all international PISA publications and sample PISA tasks for the print and computer-based assessments)

Irish PISA website: http://www.erc.ie/pisa

(includes all national PISA publications, including technical comparisons of results across cycles, and general information)

National PISA contact: pisa2012@erc.ie

1. Introduction

The OECD's Programme for International Student Assessment (PISA) is an international survey of the skills and knowledge of 15-year-old students in mathematics, reading and science. The survey, which first took place in 2000, is conducted in three-yearly cycles. In each cycle, one domain becomes the major focus of the assessment and the other domains are assessed as minor domains. Mathematics was the major focus for the second time in PISA 2012. In addition to mathematics, PISA 2012 comprised paper-based (print) assessments of reading literacy and science, and optional computer-based assessments of mathematics, reading literacy and problem-solving, with Ireland participating in all of these. This guide presents the results of the PISA 2012 mathematics assessments (print- and computer-based) and the computer-based assessment of problem solving.

In 2012, 65 countries/economies (including all 34 OECD member states) participated in PISA (Table 1.1) and all completed the print assessment of mathematics. Of the 65, 44 also took part in the computer-based assessment of problem solving, while 32 participated in the computer-based assessments of mathematics and reading. In each of these countries/economies, including Ireland, a randomly selected subset of students sampled for the print assessment was also selected to participate in the computer-based assessment.

Table 1.1. Countries/economies that participated in PISA 2012

OECD countries		Partner countries/economies	
Australia**	Japan**	Albania	Macao-China**
Austria**	Korea, Republic of**	Argentina	Malaysia*
Belgium**	Luxembourg	Brazil**	Montenegro*
Canada**	Mexico	Bulgaria*	Peru
Chile**	Netherlands*	China (Shanghai)**	Qatar
Czech Republic*	New Zealand	Chinese Taipei**	Romania
Denmark**	Norway**	Colombia**	Russian Federation**
Estonia**	Poland**	Costa Rica	Serbia, Republic of*
Finland*	Portugal**	Croatia*	Singapore**
France**	Slovak Republic**	Cyprus*	Thailand
Germany**	Slovenia**	Hong Kong-China**	Tunisia
Greece	Spain**	Indonesia	United Arab Emirates**
Hungary**	Sweden**	Jordan	Uruguay*
Iceland	Switzerland	Kazakhstan	Vietnam
Ireland**	Turkey*	Latvia	
Israel**	United Kingdom*	Liechtenstein	
Italy**	United States**	Lithuania	

* participated in the computer-based assessment of problem solving but not the computer-based assessment of mathematics and reading
** participated in the computer-based assessment of mathematics, reading and problem solving

1.1. PISA in Ireland

In Ireland, 188 schools were randomly selected, based on size, sector (secondary, community/ comprehensive, vocational), gender composition and socio-economic composition to participate in PISA. Of these, 183 schools participated in the assessment giving a weighted[1] response rate of 99.3%. In each school, up to 35 15-year-old students were randomly selected to complete the assessment.

[1] In PISA, data are weighted to ensure that population estimates are unbiased.

Overall, 5,016 students, or 84.1% of all selected students, completed the print assessment. A subsample of up to 18 students was selected to participate in the computer-based assessments in each school. A total of 2,396 students completed the computer-based assessment, which was 67% of all students sampled to participate.[2]

1.2. The Relevance and Role of PISA

PISA 2012 was the fifth cycle of the study and the second to assess mathematics in detail. In 2003, the first time mathematics was assessed as a major domain in PISA, the average mathematics score for Ireland did not differ significantly from the OECD average and Ireland ranked 17th among 29 OECD countries. This was in contrast to Ireland's relatively strong performance on the PISA reading and science assessments (Cosgrove, Shiel, Sofronoiu, Zastrutzki & Shortt, 2005). Clear differences were noted between PISA mathematics and the Junior Certificate mathematics syllabus and examination that students would have experienced at that time. In particular, it was felt that students in Ireland would be unfamiliar with many of the real-world contexts in which PISA items are presented.

These findings, as well as findings from other studies (e.g., Lyons, Lynch, Close, Sheering & Boland, 2003) contributed to the discussion on the need to reform mathematics education at post-primary level in Ireland. While the new post-primary mathematics curriculum, which was introduced to all schools through the Project Maths initiative in 2010, is not based on the PISA mathematics framework, it is more closely aligned to it than the pre-2010 junior cycle curriculum (Merriman, Shiel, Cosgrove & Perkins, 2014). The aims of Project Maths include the development of students' understanding of mathematical concepts, the development of mathematical skills and the application of knowledge and skills to solving both familiar and unfamiliar problems, using examples from everyday life which are meaningful to students (NCCA/DES, 2011a, 2011b); these aims are similar to those outlined in the PISA 2012 mathematics framework (see Chapter 2).

The focus of PISA is on what is important for citizens to know and be able to do (OECD, 2013a) and, as such, it is not designed to evaluate performance on school curricula. However, the breadth of the assessments allows for the identification of particular areas of strength or weakness in a domain. In this way PISA can be used as a tool to help identify areas of a curriculum in a country that are working well or that may benefit from revision. In Ireland in 2012, the vast majority of students who completed the PISA assessments would not have had any formal experience of the new junior cycle mathematics curriculum. However, at the time of the assessment, 23 schools had participated in the initial introduction of the Project Maths initiative which began in 2008, and all these schools were included in the sample of PISA schools for Ireland. A comparison of the mathematics performance of students in these initial schools and students in the other schools in the PISA sample has been carried out (Merriman, Shiel, Cosgrove & Perkins, 2014) and is described in Chapter 6 of this report. A national survey of mathematics teachers and mathematics co-ordinators was also conducted in Ireland as part of PISA 2012. The results of this survey, which includes teachers' views on the implementation of Project Maths, can be found in two reports: *Teaching and Learning in Project Maths: Insight from Teachers Who*

[2] Each test administrator was supplied with 15 rather than 18 laptops on which the computer-based assessment was to be carried out due to the likelihood of absences on the test day. Therefore, in some schools where more than 15 of the selected students were present on the day of the assessment a small number were unable to participate in the computer-based assessment.

Participated in PISA 2012 (Cosgrove, Perkins, Shiel, Fish & McGuinness, 2012) and *Mathematics in Transition Year: Insights of Teachers from PISA 2012* (Moran, Perkins, Cosgrove & Shiel; 2013).

There has also been reform in the wider educational context in Ireland in recent years. Both the *National Strategy to Improve Literacy and Numeracy among Children and Young People, 2011-2020* (DES, 2011) and the *Framework for Junior Cycle* (DES, 2012; 2015) propose a number of targets and reforms that have relevance for the teaching and learning of literacy and numeracy in post-primary schools. Of particular relevance to PISA are the targets in the *National Strategy* to increase the percentage of students performing at or above Level 4 on the PISA literacy and numeracy assessments by five percentage points and to halve the percentage of students performing below Level 1, relative to 2009 levels. Observing trends in PISA scores across cycles can be used as one approach to monitoring changes in achievement that may arise from curricular or educational reform. However, given the potential difficulties in estimating changes over time using assessments such as PISA, findings should be considered in the wider context and corroborating evidence sought when drawing conclusions.

A relatively recent development in PISA is the introduction of computer-based assessments of mathematics, reading and problem solving. These assessments allow us to not only assess competencies in novel ways, but can also have implications for teaching and learning within schools. Students' access to and use of technology in school could contribute towards their achievement levels on such assessments. In the future, the PISA assessments will be delivered entirely by computer.

1.3. How to Interpret the Analyses in this Report

OECD average

Throughout this report reference is made to the OECD average. This is the arithmetic mean of all OECD countries that have valid data on the indicator in question. For print mathematics, the OECD average refers to the average of the 34 OECD countries that participated in the PISA paper-based assessment. For computer-based mathematics it refers to the 23-participating OECD countries and for computer-based problem solving it refers to the 28-participating OECD countries. Where reference is made to 'OECD' in tables and figures, this always refers to the OECD average. Also in this report, 'mean' and 'average' are used interchangeably.

Comparing mean scores

Because PISA assesses samples of students, and students only attempt a subset of PISA items, achievement estimates are prone to uncertainty arising from sampling and measurement error. The precision of these estimates is measured using the standard error, which is an estimate of the degree to which a statistic, such as a country mean, may be expected to vary about the true (but unknown) population mean. Assuming a normal distribution, a 95% confidence interval can be created around a mean using the following formula: *Statistic ± 1.96 standard errors.* The confidence interval is the range in which we would expect the population estimate to fall 95% of the time, if we were to use many repeated samples. The standard errors associated with mean achievement scores in PISA were computed in a way that takes account of the two-stage, stratified sampling technique used in PISA. The approach used for calculating sampling variances for PISA estimates is known as Fay's Balanced Repeated Replication (BRR), or balanced half-samples, which takes into account the clustered nature of the sample. Using this method, half of the sample is weighted by a K factor, which must be between 0 and 1 (set at 0.5 for PISA analyses), while the other half is weighted by 2-K.

Statistical significance

Often a result is said to be statistically significant if its probability of occurrence by chance (p) is less than 0.05 (i.e. five times out of 100). In this report, mean scores are sometimes compared for countries or groups of students. When it is noted that these scores differ significantly from one another (i.e. $p<.05$), the reader can infer that the difference is *statistically* significant.

Standard deviation

The standard deviation is a measure of the spread of scores for a particular group. The smaller the standard deviation, the less dispersed the scores are. The standard deviation provides a useful way of interpreting the difference in mean scores between groups, since it corresponds to percentages of a normally distributed population, i.e., 68% of students in a population have an achievement score that is within one standard deviation of the mean and 95% have a score that is within two standard deviations of the mean.

Proficiency levels

In PISA, student performance and the level of difficulty of assessment items are placed on a single scale for each domain assessed. Using this approach means that each scale can be divided into proficiency levels and the skills and competencies of students within each proficiency level can be described. In 2012, six proficiency levels are described for mathematics and problem solving. Level 2 is considered the basic level of proficiency needed to participate effectively and productively in society and in future learning (OECD, 2013a). Within a level, all students are expected to answer at least half of the items at that level correctly (and fewer than half of the items at a higher level). A student scoring at the bottom of a proficiency level has a .62 probability of answering the easiest items at that level correctly, and a .42 probability of answering the most difficult items correctly. A student scoring at the top of a level has a .62 probability of getting the most difficult items right, and a .78 probability of getting the easiest items right.

Correlations

Correlation coefficients describe the strength of a relationship between two variables (e.g., the relationship between socio-economic status and reading achievement). However, a correlation does not necessarily imply a causal relationship. The value of a correlation (i.e. the *r* value) can range from -1 to +1. A value of 0 indicates that there is no relationship between variables, while the closer a value is to ±1, the stronger the relationship. The magnitudes of correlations are assigned qualitative labels to assist in interpretation (weak [<±.1], weak to moderate [±.1 to .25], moderate [±.25 to .4], moderate to strong [±.4 to .55]), and strong [±.55 or greater]. A negative correlation (e.g., -.26) means that as one variable increases, the other tends to decrease; a positive correlation (e.g., .26) means that both tend to increase or decrease together.

2. How does PISA Assess Mathematics and Problem Solving?

This chapter describes the PISA 2012 mathematics and problem-solving frameworks. Definitions for mathematical literacy and problem solving are provided and the foundations of the frameworks are described. The characteristics of mathematics (print and computer-based) and computer-based problem-solving assessments are also described. Sample items from the PISA 2012 assessment can be found in Chapter 3.

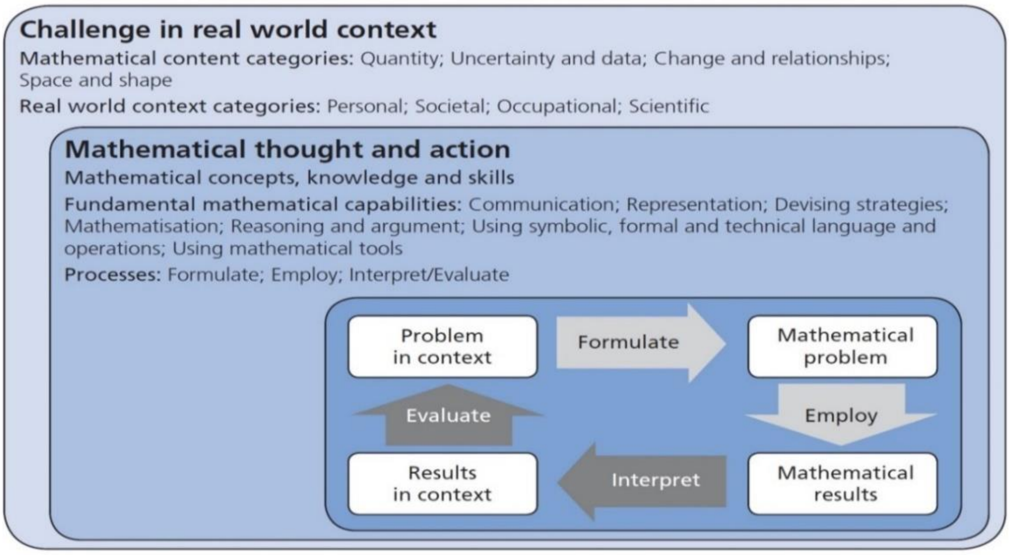
2.1. The PISA 2012 Mathematics Framework

PISA 2012 (OECD, 2013a, p. 25) defines mathematical literacy as:

> An individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematics concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

Mathematical modelling is central to the PISA definition of mathematical literacy and is illustrated in Figure 2.1. The mathematical modelling process begins with a **problem situated in a real-life context**. To solve the problem, the situation is **formulated** mathematically by drawing upon a variety of mathematical *concepts, knowledge and skills*; by making use of fundamental mathematical *capabilities*; and by engaging in different mathematical *processes*. Thus the problem is transformed into a **mathematical problem.** The problem-solver then **employs** mathematical capabilities and processes to obtain **mathematical results**. The mathematical results must then be **interpreted** and **evaluated** in terms of the **original contextual problem**. Depending on the nature of the mathematical problem to be solved, it may not be necessary to engage in all stages of the modelling cycle, and many PISA items involve only parts of the cycle.

Figure 2.1. A model of PISA mathematical literacy in practice (OECD, 2013a)



In the PISA framework, the real-life problem is characterised in terms of the **mathematical content** that underlies the challenge and the real-world **context** in which it arises, as well as the three key **mathematical processes** underlying the solution.

2.1.1. Mathematical Content Knowledge

Four mathematical content categories, which are not intended to be mutually exclusive, are specified in the PISA mathematics framework (OECD, 2013a):

- ***Change & Relationships***, which involves understanding types of change and recognising when they occur in order to use suitable mathematical models to describe and predict change.
- ***Space & Shape***, which involves understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. This category draws on geometry, spatial visualisation, measurement and algebra.
- ***Quantity***, which involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results.
- ***Uncertainty & Data***, which involves knowledge of variation in processes, uncertainty and error in measurement and chance.

The PISA mathematics assessment includes a range of content topics: functions; algebraic expressions; equations and inequalities; co-ordinate systems; relationships within and among geometrical objects in two and three dimensions; measurement; numbers and units; arithmetic operations; percentages, ratios and proportions; counting principles; estimation; data collection; data variability and its description; samples and sampling; and chance and probability (OECD, 2013a).

2.1.2. Mathematical Contexts

The PISA tests are designed to ensure that the selection of assessment items reflects a broad range of settings. Four context categories are outlined in the PISA framework: personal, occupational, societal, and scientific.

2.1.3. Mathematical Processes and the Underlying Mathematical Capabilities

The PISA 2012 definition of mathematical literacy refers to three processes that correspond to the different stages of the mathematical modelling cycle. These are:

- ***Formulating*** situations mathematically;
- ***Employing*** mathematical concepts, facts, procedures, and reasoning; and
- ***Interpreting***, applying, and evaluating mathematical outcomes.

Seven fundamental mathematical capabilities are identified as underpinning these processes: communication; mathematising; representation; reasoning and argument; devising strategies for solving problems; using symbolic, formal and technical language and operations; and using mathematical tools (OECD, 2013a).

2.1.4. Computer-based Assessment of Mathematics

An optional computer-based assessment of mathematics was included for the first time in PISA 2012. In Ireland, a subset of students who sat the print assessment also took the computer-based assessment. The computer-based assessment of mathematics is underpinned by the same framework as the print mathematics assessment.

The items that make up the computer-based assessment of mathematics are described in terms of three aspects:

1. The ***mathematical competencies*** being tested, i.e. the aspects of mathematical literacy which are present in all environments, not just computer environments. One or more of these are tested in every computer-based mathematics item, with the main one used for classification purposes.
2. ***Competencies that cover aspects of mathematics and ICT***, such as: making a chart from data; producing graphs of functions; using graphs to answer questions about functions; sorting information and planning efficient sorting strategies; using virtual instruments such as an on-screen ruler or protractor; and transforming images using a dialog box or mouse. In an effort to separate the effects of this type of item format on performance, these competencies are assessed in some items only.
3. ***ICT skills***, i.e. the fundamental skills needed to work with a computer, including basic knowledge of hardware (e.g. keyboard and mouse) and of conventions (e.g. arrows to move). Items were designed to keep the need for such skills to a minimum core level.

2.1.5. PISA 2012 Mathematics Test Characteristics

The print mathematics test consists of 110 items, while the computer-based mathematics assessment contains 41 items. The PISA 2012 mathematics test items are classified according to the main elements of the framework as outlined above (Table 2.1). With regard to the mathematical processes, about half of items belong to the process *employing mathematical concepts, facts, procedures, and reasoning,* while the remainder of the items are split approximately evenly between the two processes that involve *formulating situations mathematically* and *interpreting, applying, and evaluating mathematical outcomes*. Items are distributed approximately evenly across the content and context categories.

Table 2.1. Distribution of 2012 mathematics items by process, content, and context

Process	%	Content	%	Context	%
Print-based Assessment					
Formulating situations mathematically	29.3	Change & Relationships	26.6	Personal	19.3
Employing mathematical concepts, facts, procedures, and reasoning	45.9	Space & Shape	24.8	Occupational	22.0
Interpreting, applying and evaluating mathematical outcomes	24.8	Quantity	25.7	Societal	33.0
		Uncertainty & Data	22.9	Scientific	25.7
Computer-based Assessment					
Formulating situations mathematically	22.0	Change & Relationships	26.8	Personal	31.7
Employing mathematical concepts, facts, procedures, and reasoning	53.6	Space & Shape	29.2	Occupational	22.0
		Quantity	22.0	Societal	26.8
Interpreting, applying and evaluating mathematical outcomes	24.4	Uncertainty & Data	22.0	Scientific	19.5

Of the print mathematics items, approximately 41% of which were multiple-choice or complex multiple-choice, 30% required a short written response, and 28% required a longer written response. Approximately 29% of computer-based mathematics items were classified as multiple-choice or complex multiple-choice, 61% as short constructed-response, and 10% as open constructed-response.

2.2. The PISA 2012 Problem-solving Framework

Problem solving was assessed as part of the computer-based assessment in PISA 2012. The assessment of problem solving has been significantly revised since it was last administered in PISA 2003, both in terms of its format and scope. The assessment has moved from a paper-based format to a computer-based platform, making possible the inclusion of items that require the student to explore the problem situation and therefore creating a greater interaction between the test-taker and the problem. Also, while PISA 2003 tested cross-disciplinary problem solving, the 2012 assessment is based on the view that solving real-life problems often involves interacting with new systems rather than applying prior knowledge and therefore excludes problems requiring expert knowledge (OECD, 2013a).

For the purposes of PISA 2012 (OECD, 2013a, p.122), problem-solving competency is defined as:

> An individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen.

This definition recognises not only the cognitive but also the motivational and affective aspects of problem solving (Funke, 2010). The PISA 2012 problem-solving framework is composed of three main elements: the *problem context*, the nature of the *problem situation;* and the *problem-solving processes*.

2.2.1. Problem Context

The problem-solving contexts in PISA are classified according to the setting (i.e., whether it involves technology or not) and the focus (personal or social).

- ***Setting***: Problems set in a *technology* context have the functionality of a technological device as their basis, e.g., mobile phones, remote controls for appliances, ticket vending machines. Problems that occur in other settings are classified as having *non-technology* contexts, e.g., route planning, task scheduling, decision-making.
- ***Focus***: *Personal* contexts include those relating to the self, family and peer groups, while *social* contexts refer to the community or society in general.

2.2.2. Nature of the Problem Situation

In the PISA assessment, two problem-solving situations have been classified: static problem situations (where the information provided to the problem solver at the outset is complete) and interactive problem situations (where it is necessary to explore the problem situation to uncover additional relevant information).

2.2.3. Problem-solving Processes

The assessment framework specifies four processes involved in problem solving:

- ***Exploring and Understanding,*** which involves building mental representations of each of the pieces of information presented in the problem, including exploring the problem situation and understanding given information and information discovered while interacting with the problem situation.
- ***Representing and Formulating***, which involves selecting relevant information and mentally organising and integrating this information with relevant prior knowledge.
- ***Planning and Executing***, which consists of goal setting; devising a plan or strategy to reach the goal; and carrying out the plan.
- ***Monitoring and Reflecting***, which includes monitoring progress towards the goal at each stage; reflecting on solutions from different perspectives; critically evaluating assumptions and alternative solutions; identifying the need for additional information or clarification and communicating progress in a suitable manner.

Engaging in these problem-solving processes requires the use of reasoning skills, such as deductive, inductive, quantitative, correlational, analogical, combinatorial and multidimensional reasoning (OECD, 2013a). A broad mix of reasoning skills is sampled across assessment items, as the complexity of the problem and types of reasoning involved affects item difficulty.

2.2.4. PISA 2012 Problem-solving Test Characteristics

The computer-based assessment of problem solving consists of 42 items distributed over 16 units, the characteristics of which are derived from the main elements of the framework (Table 2.2). There is an even split of items presented in technology and non-technology settings, while just over half of items are presented in a personal setting, with the remainder presented in a social setting. Almost two-thirds of items are considered to be interactive, with just over a third considered to be static. Almost a quarter of items are 'exploring and understanding' tasks, while just over a fifth are 'representing and formulating' tasks. Approximately 38% of items mainly involve 'planning and executing' and the remaining 17% are classified as 'monitoring and reflecting' tasks.

Table 2.2. Distribution of the 2012 problem-solving items by context (setting and focus), nature of problem situation and problem-solving process involved

Context (setting)	%	Context (focus)	%	Nature of problem situation	%	Problem-solving process	%
Technology	50.0	Social	45.2	Static	35.7	Exploring & understanding	23.8
Non-technology	50.0	Personal	54.8	Interactive	64.3	Representing & formulating	21.4
						Planning & executing	38.1
						Monitoring & reflecting	16.7

2.3. Rotated Test Design

PISA uses a rotated test design. This means that, for the print assessment, each participating student was given one of 13 possible test booklets at random. Each booklet was made up of four half-hour blocks of about 14 items. All print booklets contained some mathematics blocks, nine contained some reading blocks and nine contained some science blocks. Each student participating in the computer-based assessment was given one of 32 possible forms for the computer-based assessment. Each form of the computer-based assessment consisted of two 20-minute blocks.

Twelve forms contained only problem-solving items, four contained only reading and four were made up of only mathematics items. The remaining forms contained a combination of problem-solving, mathematics and reading items. For both the print and the computer-based assessments, blocks were repeated across booklets and forms in different positions. This allowed for a broad coverage of an assessment domain and also meant that each student's achievement score for a domain could be placed in the same scale. It also controlled for the possible confounding effect of test fatigue on students' responses.

Summary

- This chapter outlined the PISA 2012 frameworks for mathematics and problem solving.
- PISA mathematics is described in terms of the mathematical content that underlines the challenge, the mathematical processes involved in solving the problem and the real-world context of the problem.
- Four mathematical processes are specified: Change & Relationships; Space & Shape; Quantity; and Uncertainty & Data.
- Three mathematical processes are described: Formulating, Employing and Interpreting.
- Four context categories are outlined: personal, occupational, societal and scientific.
- The computer-based assessment of mathematics is underpinned by the same framework as the print mathematics assessment. The items that make up the computer-based assessment of mathematics are described in terms of the mathematical competencies being tested; competencies that cover aspects of mathematics and ICT; and ICT skills.
- The framework for the computer-based assessment of problem solving is composed of three main elements: the problem context, the nature of the problem situation and the problem-solving processes.
- The problem-solving contexts in PISA are classified according to setting (technological or not) and the focus (personal or social).
- Two problem-solving situations have been classified: static and interactive.
- The problem-solving framework specifies four processes involved in problem solving: Exploring and Understanding; Representing and Formulating; Planning and Executing; Monitoring and Reflecting.
- The mathematics and problem-solving items are classified according to the main elements of the frameworks outlined above.

3. What are the PISA Test Items Like?

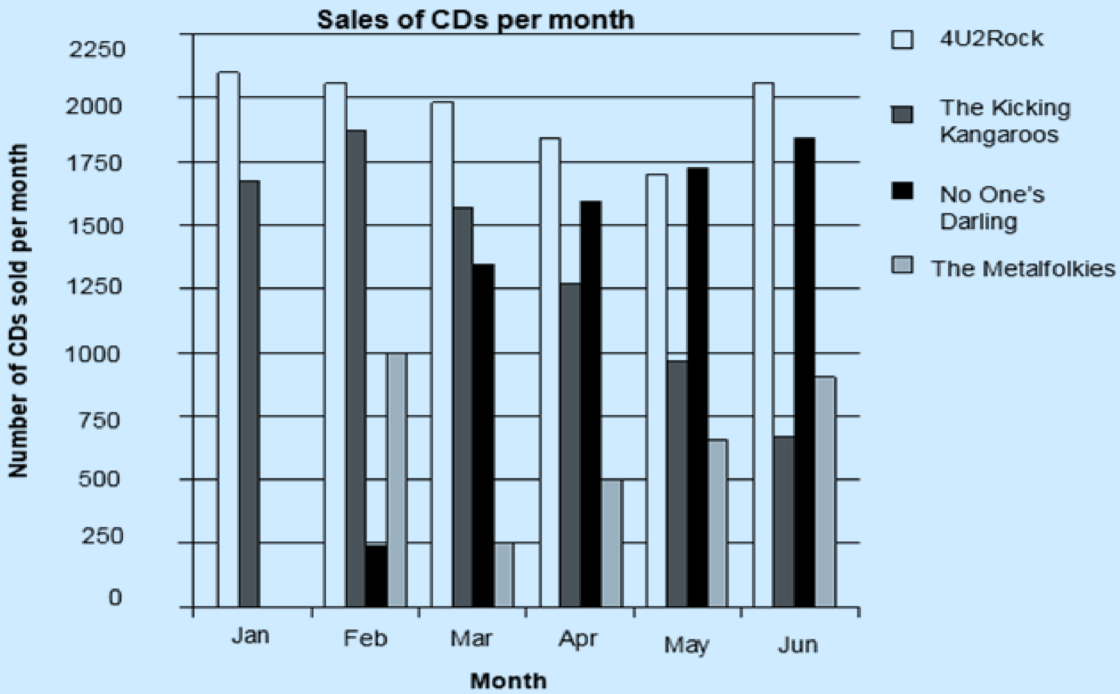
This chapter presents some sample items from the PISA 2012 mathematics (print and computer-based) and problem-solving assessments. More sample items from the PISA assessments are available at http://www.oecd.org/pisa/test.

3.1. Print Mathematics Test Items

Items from three print mathematical units are presented in this section. The percentage of students who answered each item correctly, incorrectly or missed or skipped the item in Ireland and across OECD countries, as well as the scale score and proficiency level follows each item. The mathematical content area, context and cognitive process are also identified for each item.

PRINT MATHEMATICS 1: *Charts*

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Charts – Question 3

The manager of *The Kicking Kangaroos* is worried because the number of their CDs that sold decreased from February to June.

What is the estimate of their sales volume for July if the same negative trend continues?

A→ 70 CDs
B→ 370 CDs
C→ 670 CDs
D→ 1340 CDs

Table 3.1. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Charts Question 3

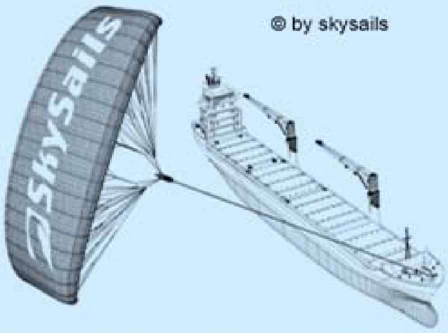
Response	Ireland	OECD	Item difficulty
Correct *(option B)*	76.2	76.7	Scale Score: 428 Proficiency Level 2
Incorrect	23.6	23.3	
Missing/Not reached	0.2	0.0	

Mathematical content area: *Uncertainty & Data*
Context: *Societal*
Process: *Employ*

PRINT MATHEMATICS 2: *Sailing Ships*

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.



Sailing Ships – Question 3

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship *NewWave* are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: *NewWave*

Type: freighter

Length: 117 metres

Breadth: 18 metres

Load capacity: 12 000 tons

Maximum speed: 19 knots

Diesel consumption per year without a kite sail: approximately 3 500 000 litres

The cost of equipping the *NewWave* with a kite sail is 2 500 000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

....

....

....

....

....

Number of years: ....

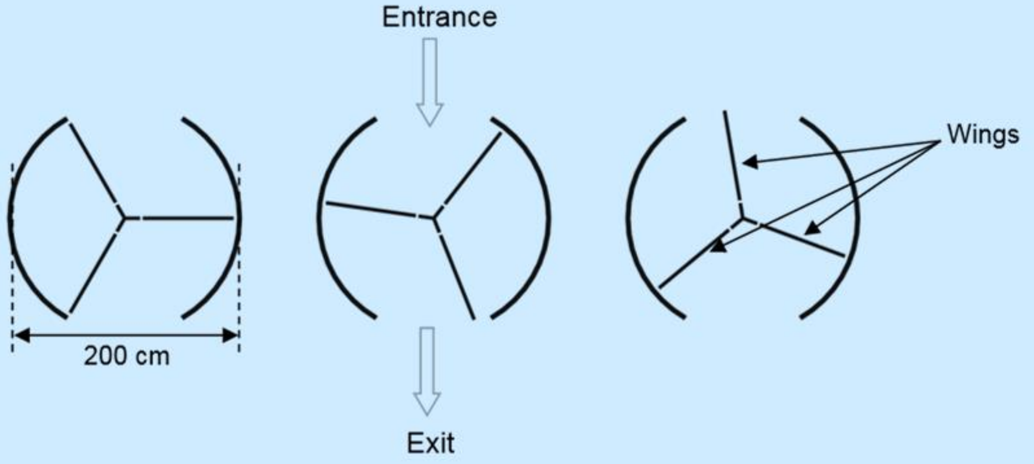
Table 3.2. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Sailing Ships Question 3

Response	Ireland	OECD	Item difficulty
Correct *(a solution from 8 to 9 years is provided with adequate mathematical calculations)*	15.8	15.3	Scale Score: 702 Proficiency Level 6
Incorrect	65.5	53.1	
Missing/Not reached	18.7	31.7	

Mathematical content area: *Change & relationships*
Context: *Scientific*
Process: *Formulate*

PRINT MATHEMATICS UNIT 3: *Revolving Door*

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Revolving Door – Question 1

What is the size in degrees of the angle formed by two door wings?

Size of the angle: ……………………………°

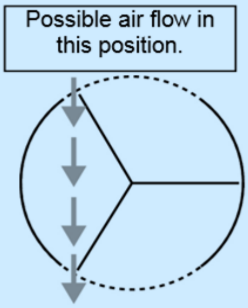
Table 3.3. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Revolving Door Question 1

Response	Ireland	OECD	Item difficulty
Correct *(120 degrees)*	63.4	57.7	Scale Score: 512 Proficiency Level 3
Incorrect	30.1	32.8	
Missing/Not reached	6.5	9.5	

Mathematical content area: *Space & Shape*
Context: *Scientific*
Process: *Employ*

Revolving Door – Question 2

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

Table 3.4. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Revolving Door Question 2

Response	Ireland	OECD	Item difficulty
Correct *(answers in the range from 103 to 105)*	2.4	3.5	Scale Score: 840 Proficiency Level 6
Incorrect	76.0	69.6	
Missing/Not reached	21.6	26.9	

Mathematical content area: *Space & Shape*
Context: *Scientific*
Process: *Formulate*

Revolving Door – Question 3

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.

What is the maximum number of people that can enter the building through the door in 30 minutes?

A - 60
B - 180
C - 240
D - 720

Table 3.5. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Revolving Door Question 3

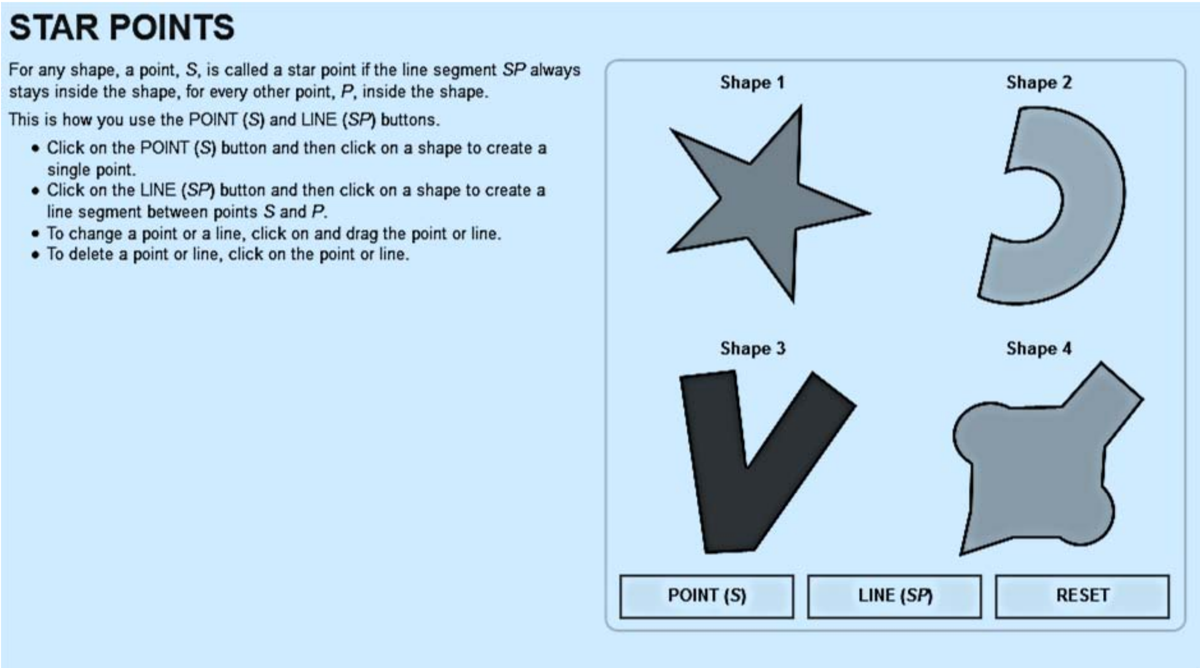
Response	Ireland	OECD	Item difficulty
Correct *(option D)*	48.8	46.4	Scale Score: 561 Proficiency Level 4
Incorrect	48.9	50.3	
Missing/Not reached	2.3	3.3	

Mathematical content area: *Quantity*
Context: *Scientific*
Process: *Formulate*

3.2. Computer-based Mathematics Test Items

Two items from the computer-based mathematics assessment are presented in this section. The percentage of students who answered each item correctly, incorrectly or missed or skipped the item in Ireland and across OECD countries follows each item. The mathematical content area, context and cognitive process are identified for each item.

COMPUTER-BASED MATHEMATICS 1: *Star Points*



Question 2: STAR POINTS *CM020Q02*

Some shapes have many star points and some have no star points.

For one of the shapes above, it is impossible to find a star point. Which one of the shapes has no star point?

- Shape 1
- Shape 2
- Shape 3
- Shape 4

Table 3.6. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Star Points Question 2

Response	Ireland	OECD	Item difficulty
Correct *(shape 2)*	44.6	47.4	Scale Score: 549 Proficiency Level 4
Incorrect	52.6	48.2	
Missing/Not reached	2.8	4.4	

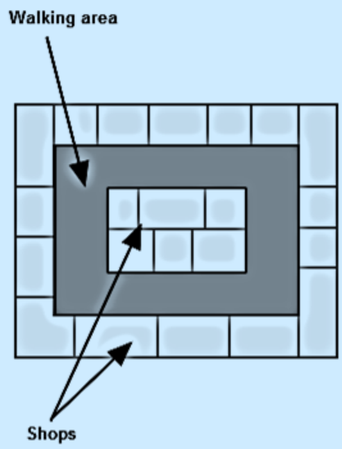
Mathematical content area: *Space & Shape*
Context*:* *Scientific*
Process: *Employ*

STAR POINTS

For any shape, a point, *S*, is called a star point if the line segment *SP* always stays inside the shape, for every other point, *P*, inside the shape.

This is how you use the POINT (*S*) and LINE (*SP*) buttons.

- Click on the POINT (*S*) button and then click on a shape to create a single point.
- Click on the LINE (*SP*) button and then click on a shape to create a line segment between points *S* and *P*.
- To change a point or a line, click on and drag the point or line.
- To delete a point or line, click on the point or line.



POINT (*S*) | LINE (*SP*) | RESET

Question 3: STAR POINTS *CM020Q03*

Shown above is a floor plan design for a new single-level shopping centre. The coloured region represents areas where people can walk.

Security cameras are to be installed to observe all of the walking area. The security cameras will be mounted on the ceiling of the walking area, where they can view 360° and cover the visible walking area.

Place star points on the plan above to show where security cameras should be installed to meet the following conditions:

- Each part of the walking area can be observed from at least one camera.
- The **fewest** number of cameras is used.

Table 3.7. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Star Points Question 3

Response	Ireland	OECD	Item difficulty
Correct *(diagonally opposite corners of the shaded area)*	18.7	26.9	Scale Score: 644 Proficiency Level 5
Incorrect	73.8	63.5	
Missing/Not reached	7.5	9.6	

Mathematical content area: *Space & Shape*
Context: *Scientific*
Process: *Employ*

COMPUTER-BASED MATHEMATICS 2: *Body Mass Index*

BODY MASS INDEX

Tegan and Raul are doing a project about body mass and health.

They find the Zedhealth website about health and Body Mass Index (BMI).

Another web page had some data about Zedland's obesity rates.

Zedhealth - Web Browser

http://www.zedhealth.gov.zd/BMIstats

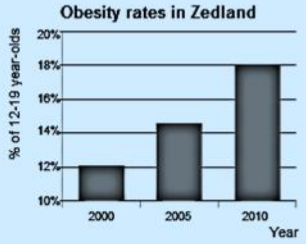
Your BMI | Statistics | Zedland Data

Obesity Rates in Zedland

The table shows the percentage of 12 to 19 year-olds in Zedland in each of the four BMI classifications in 2000, 2005 and 2010.

The chart illustrates the rates of obesity for each of the three years.

BMI Classification:	Percentage of 12-19 year-olds		
	2000	2005	2010
Underweight	14.1%	14.8%	14.3%
Normal	57.5%	53.5%	51.5%
Overweight	16.3%	17.1%	16.1%
Obese	12.1%	14.6%	18.1%



Question 2: BODY MASS INDEX *CM038Q05*

What is one major change in the BMI classifications for 12-19 year-olds in Zedland between 2000 and 2010? Justify your answer based on value(s) from the data table.

Table 3.8. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Body Mass Index Question 2

Response	Ireland	OECD	Item difficulty
Correct *(a statement that makes a correct reference to both the size and direction of the change)*	38.8	27.8	Scale Score: 641
Incorrect	50.1	54.5	Proficiency Level 5
Missing/Not reached	11.1	17.7	

Mathematical content area: *Uncertainty & Data*
Context*: Societal*
Process: *Interpret*

Question 3: BODY MASS INDEX *CM038Q06*

Tegan says: "The graph shows that the obesity rate in 2010 is about four times the rate that it was in 2000."

Tegan is not correct. Explain what is misleading about the graph that may have led Tegan to draw this conclusion

Table 3.9. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Body Mass Index Question 3

Response	Ireland	OECD	Item difficulty
Correct *(an explanation that the graph is misleading because either (i) the scale of the vertical axis does not start at zero or (ii) that the visual impression of the graph's bar heights does not match the numerical values from the table or the vertical axis of the graph)*	26.0	23.2	Scale Score: 660 Proficiency Level 5
Incorrect	64.2	60.2	
Missing/Not reached	9.8	16.6	

Mathematical content area: *Uncertainty & Data*
Context*:* *Societal*
Process: *Interpret*

3.3. Computer-based Problem-solving Test Items

Three items from one problem-solving unit are presented in this section. The percentage of students who answered each item correctly, incorrectly or missed or skipped the item in Ireland and across participating OECD countries, as well as the scale score and proficiency level follows each item. The problem-solving context, nature of the problem and cognitive process are identified for each item.

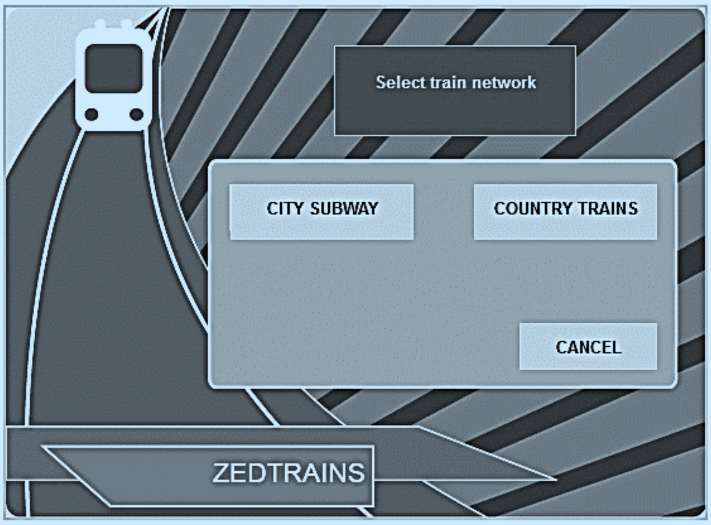
COMPUTER-BASED PROBLEM SOLVING 1: *Tickets*

TICKETS

A train station has an automated ticketing machine. You use the touch screen on the right to buy a ticket. You must make three choices.

- Choose the train network you want (subway or country).
- Choose the type of fare (full or concession).
- Choose a daily ticket or a ticket for a specified number of trips. Daily tickets give you unlimited travel on the day of purchase. If you buy a ticket with a specified number of trips, you can use the trips on different days.

The BUY button appears when you have made these three choices. There is a CANCEL button that can be used at any time BEFORE you press the BUY button.



Question 1: TICKETS *CP038Q02*

Buy a full fare, country train ticket with two individual trips.

Once you have pressed BUY, you cannot return to the question.

Table 3.10. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Tickets Question 1

Response	Ireland	OECD	Item difficulty
Correct *(students must first select 'country trains', then the fare type ['full fare'], then choose between a daily ticket and one for multiple trips and finally indicate the number of trips [two])*	66.3	58.0	Scale Score: 526 Proficiency Level 3
Incorrect	32.9	40.3	
Missing/Not reached	0.8	1.7	

Context: *Technology/social*
Nature: *Interactive*
Process: *Planning and Executing*

Question 2: TICKETS *CP038Q01*

You plan to take four trips around the city on the subway today. You are a student, so you can use concession fares.

Use the ticketing machine to find the cheapest ticket and press BUY.

Once you have pressed BUY, you cannot return to the question.

Table 3.11. Percent fully correct, partially correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Tickets Question 2

Response	Ireland	OECD	Item difficulty
Full credit *(students must compare the daily subway tickets with concession fares and the individual concession fare ticket with four trip screen before buying the least expensive ticket)*	28.7	27.0	Scale Score: 638 Proficiency Level 5
Partial credit *(students who buy one of the two tickets without comparing the prices for the two only)*	46.1	46.3	Scale Score: 453 Proficiency Level 2
Incorrect	24.2	24.1	
Missing/Not reached	1.0	2.6	

Context: *Technology/Social*
Nature: *Interactive*
Process: *Exploring and Understanding*

Question 3: TICKETS *CP038Q03*

You want to buy a ticket with two individual trips for the city subway. You are a student, so you can use concession fares.

Use the ticketing machine to purchase the best ticket available.

Table 3.12. Percent correct, incorrect and missing or not reached for students in Ireland and across OECD countries and item difficulty for Tickets Question 3

Response	Ireland	OECD	Item difficulty
Correct *(students must follow steps to buy tickets. When concession fares are selected the machine says that 'there are no tickets of this type available'. Students must adjust their initial plan by buying a full fare ticket for the subway)*	52.7	42.9	Scale Score: 579 Proficiency Level 4
Incorrect	45.9	55.3	
Missing/Not reached	1.4	1.8	

Context: *Technology/Social*
Nature: *Interactive*
Process: *Monitoring and Reflecting*

4. How did Students in Ireland Perform?

Each student participating in the PISA study responded to items in one of 13 test booklets. Items in each domain were repeated across booklets in such a way that it was possible to place each student's performance on the same overall scale. Each student's score is based on the difficulty of the tasks they answer correctly. The scale for each domain is constructed so that the mean across OECD countries is 500 and the standard deviation is 100. The mathematics scale was constructed in 2003, when mathematics was first assessed as a major domain.[3] As the computer-based assessment of problem solving was a new assessment in 2012 a new scale was created for this domain.

4.1. Performance on Print Mathematics

Table 4.1 shows the average scores of all countries/economies that participated in PISA 2012 and their position compared to the OECD mean score (494). Students in Ireland achieved a mean score of 501 on the overall print mathematics scale, which is significantly above the OECD mean score of 494. Ireland's performance is ranked 13th out of 34 OECD countries and 20th out of all 65 participating countries/economies. When measurement and sampling error are accounted for (applying a 95% confidence interval) Ireland's true rank ranges from 11th to 17th among OECD countries and from 18th to 24th among all participating countries/economies.

Table 4.1. Mean country/economy scores for the print mathematics scale and positions relative to the OECD mean score

Above OECD mean		**Same as OECD mean**	**Below OECD mean**	
Shanghai-China *(613)*	Vietnam *(511)*	Czech Republic *(499)*	Luxembourg *(490)*	Kazakhstan *(432)*
Singapore *(573)*	Austria *(506)*	France *(495)*	Italy *(485)*	Thailand *(427)*
Hong Kong-China *(561)*	Australia *(504)*	United Kingdom *(494)*	Spain *(484)*	Chile *(423)*
Chinese Taipei *(560)*	**Ireland *(501)***	Iceland *(493)*	Russian Fed. *(482)*	Malaysia *(421)*
Korea *(554)*	Slovenia *(501)*	Latvia *(491)*	Slovak Republic *(482)*	Mexico *(413)*
Macao-China *(538)*	Denmark *(500)*	Norway *(489)*	United States *(481)*	Montenegro *(410)*
Japan *(536)*	New Zealand *(500)*	Portugal *(487)*	Lithuania *(479)*	Uruguay *(409)*
Liechtenstein *(535)*	Australia *(504)*		Sweden *(478)*	Costa Rica *(407)*
Switzerland *(531)*	Ireland *(502)*		Hungary *(477)*	Albania *(394)*
Netherlands *(523)*	Slovenia *(501)*		Croatia *(471)*	Brazil *(391)*
Estonia *(521)*	Denmark *(500)*		Israel *(466)*	Argentina *(388)*
Finland *(519)*	New Zealand *(500)*		Greece *(453)*	Tunisia *(388)*
Canada *(518)*			Serbia *(449)*	Jordan *(386)*
Poland *(518)*			Turkey *(448)*	Colombia *(376)*
Belgium *(515)*			Romania *(445)*	Qatar *(376)*
Germany *(514)*			Cyprus *(440)*	Indonesia *(375)*
			Bulgaria *(439)*	Peru *(368)*
			UAE *(434)*	

Source: OECD (2013b)

[3] In each cycle of PISA one subject area or domain is assessed as the major domain and the other domains are assessed as minor domains and therefore receive less emphasis.

4.1.1. Performance of Higher- and Lower-achieving Students on Print Mathematics

The performance of higher- and lower-achieving students in PISA can be observed by examining the performance of students at the 10th and 90th percentiles. The score achieved by students in Ireland at the 10th percentile (391) is significantly above the corresponding OECD average score (375). On the other hand, the score of students in Ireland at the 90th percentile (610) does not differ significantly from the corresponding average score across OECD countries (614). This means that lower-achieving students in Ireland are doing relatively better than lower-achieving students on average across OECD countries, while higher-achieving students in Ireland are performing at about the average level for higher-achieving students across OECD countries.

PISA also describes performance in terms of proficiency levels. Scores on the print mathematics scale are grouped into six levels of proficiency, each characterised by different levels of skills and knowledge (Table 4.2). Level 6 is the highest proficiency level and describes the skills of students who are able to complete the most difficult PISA items successfully, while Level 1 is the lowest level and students performing at or below this level are likely to correctly answer only the easiest PISA items. In PISA, Level 2 is considered the baseline level of proficiency that is required to participate fully in society and future learning (OECD, 2013b).

In Ireland, 17% of students are performing below Level 2 on the print mathematics scale, which is significantly lower than the OECD average of 23%. The proportion of students in Ireland performing at the highest proficiency levels (Level 5 and 6) is also significantly lower than the corresponding OECD average (11% and 13%, respectively). These outcomes are broadly consistent with those based on performance at the 10th and 90th percentiles.

Table 4.2. Descriptions of the six levels of proficiency on the overall print and computer-based mathematics scales

Level *(Cut-point)*	**Students at this level are capable of:**
6 *(669 and above)*	Conceptualising, generalising and using information based on their investigations and modelling of complex problem situations; using knowledge in relatively non-standard contexts; linking different information sources and representations and moving flexibly among them; applying their insight and understanding, along with mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations; reflecting on their actions and formulating and precisely communicating their actions and reflections regarding their findings, interpretations and arguments, and explaining why they were applied to the original situation. Students at this level are able to successfully complete the most difficult PISA items.
5 *(607 to less than 669)*	Developing and working with models of complex situations, including identifying constraints and specifying assumptions; selecting, comparing and evaluating appropriate problem-solving strategies for dealing with complex problems related to these models; working strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations and insights pertaining to these situations; beginning to reflect on their work and formulating and communicating their interpretations and reasoning.
4 *(545 to less than 607)*	Working effectively with explicit models of complex, concrete situations that may involve constraints or making assumptions; selecting and integrating different representations (including symbolic representations) and linking them directly to aspects of real-world situations; using their limited range of skills and reasoning with some insight in straightforward contexts; constructing and communicating explanations and arguments based on their interpretations, arguments and actions.
3 *(482 to less than 545)*	Executing clearly described procedures (including those that require sequential decisions); making sufficiently sound interpretations to be able to build simple models or select and apply simple problem-solving strategies; interpreting and using representations based on different information sources and reasoning directly from them; handling percentages, fractions and decimal numbers and working with proportional relationships; engaging in basic interpretation and reasoning.
2 *(420 to less than 482)*	Interpreting and recognising situations in contexts that require no more than direct inference; extracting relevant information from a single source and making use of a single representational mode; employing basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers; making literal interpretations of results. Level 2 is considered the baseline level of mathematical proficiency that is required to participate fully in modern society.
1 *(358 to less than 420)*	Answering questions involving familiar contexts where all relevant information is present and the questions are clearly defined; identifying information and carrying out routine procedures according to direct instructions in explicit situations; performing actions that are almost always obvious and follow immediately from the given stimuli.
Below Level 1 *(below 358)*	Performing very direct and straightforward mathematical tasks, such as reading a single value from a well-labelled chart or table where the labels on the chart match the words in the stimulus and question, so that the selection criteria are clear and the relationship between the chart and the aspects of the contexts depicted are evident; performing arithmetic calculations with whole numbers by following clear and well-defined instructions.

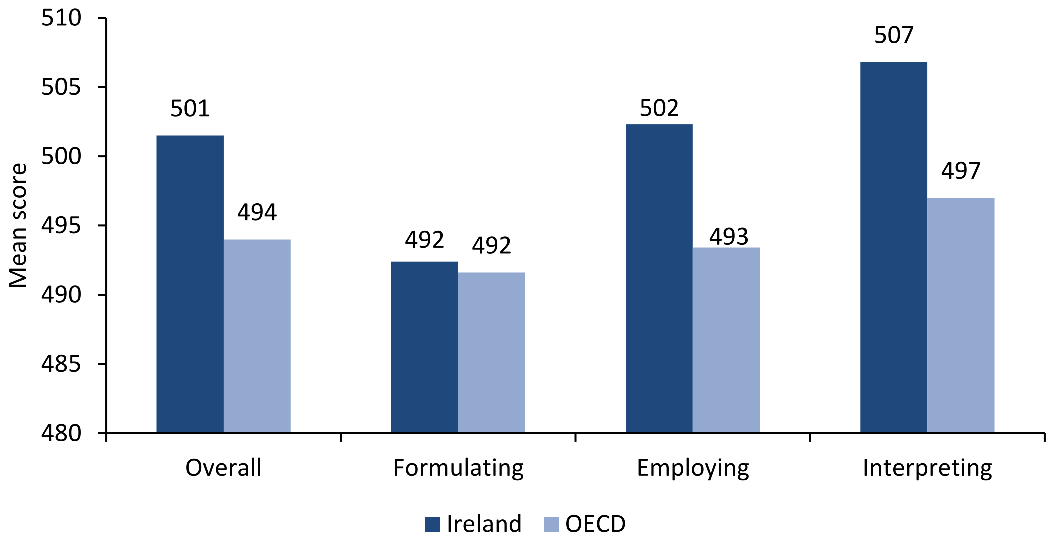
Source: OECD (2013b)

4.1.2. Performance of Students on the Print Mathematics Subscales

PISA also reports student performance according to three mathematical processes (Formulating, Employing and Interpreting) and four mathematical content subscales (Change & Relationships, Space & Shape, Quantity, and Uncertainty & Data). The PISA mathematical literacy framework, outlined in Chapter 2, describes the tasks involved in each of these scales.

Figure 4.1 presents the mean scores for students in Ireland and the corresponding OECD average scores for the overall print mathematics scale and each of the three process subscales. The performance of students in Ireland on both the Employing and Interpreting subscales is significantly above the corresponding OECD averages. On the other hand, the mean score for students in Ireland on the Formulating subscale (492.4) does not differ significantly from the average across OECD countries (491.6).

Figure 4.1. Mean scores on the overall mathematics scale and the three mathematical process subscales for print mathematics, in Ireland and on average across OECD countries



Source: OECD (2013b).

Ireland has fewer lower-performing students (below Level 2) on each of the process subscales compared to the corresponding OECD averages (Table 4.3). On the other hand, the proportions of higher-achieving students in Ireland on each of the process subscales are slightly lower than the corresponding OECD averages.

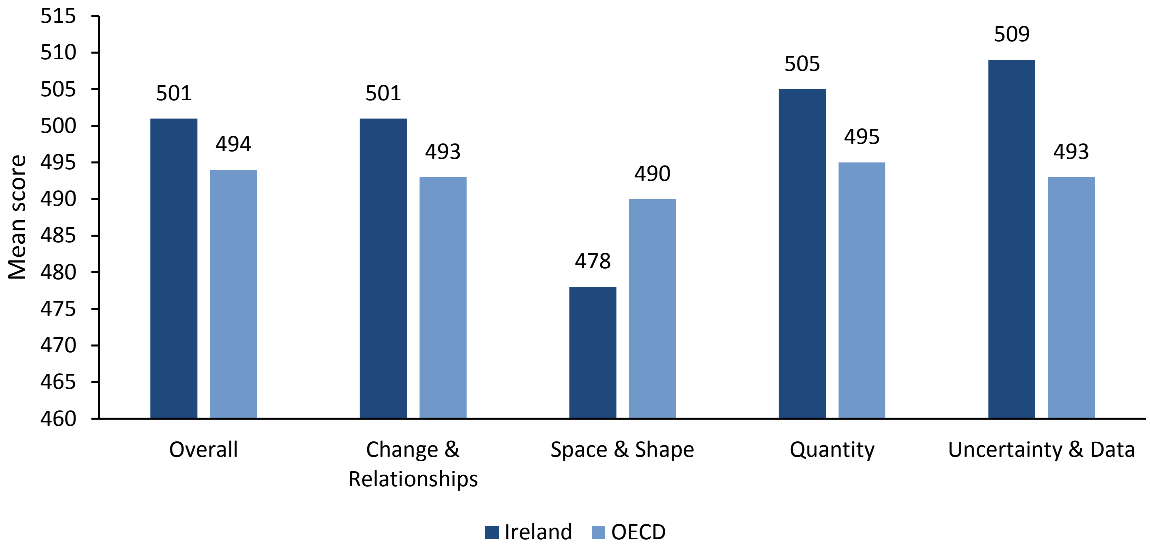
Table 4.3. Percentages of students below Level 2 and at Level 5 or above for the three print mathematical processes subscales, in Ireland and on average across OECD countries

	Formulating		Employing		Interpreting	
	Below Level 2	At or above Level 5	Below Level 2	At or above Level 5	Below Level 2	At or above Level 5
Ireland	23%	12%	16%	10%	17%	13%
OECD	26%	15%	23%	12%	23%	14%

Source: OECD (2013b)

The mean scores of students in Ireland on the four content subscales are presented with the corresponding OECD averge scores in Figure 4.2. Students in Ireland have significantly higher mean scores on the Change & Relationships (501), Quantity (505) and Uncertainty & Data (509) subscales compared to the OECD average scores (493, 495 and 493, respectively); however, they perform significantly less well on the Space & Shape subscale (478 compared to 490).

Figure 4.2. Mean scores on the overall mathematics scale and the four mathematical content area subscales for print mathematics, in Ireland and on average across OECD countries



Source: OECD (2013b)

Ireland has fewer lower-performing (i.e., below Level 2) students on each of the content area subscales compared to the corresponding OECD averages, with the exception of the Space & Shape subscale (Table 4.4). The proportions of higher-performing students (i.e. at Level 5 or above) in Ireland on the Change & Relationships and Space & Shape subscales are lower than the corresponding OECD averages. On the other hand, the proportions of higher-performing students on the Quantity and Uncertainty & Data subscales are similar to the corresponding OECD average proportions.

Table 4.4. Percentages of students below Level 2 and at Level 5 or above for the four print mathematical content area subscales, in Ireland and on average across OECD countries

	Change & Relationships		Space & Shape		Quantity		Uncertainty & Data	
	Below Level 2	At or above Level 5	Below Level 2	At or above Level 5	Below Level 2	At or above Level 5	Below Level 2	At or above Level 5
Ireland	18%	11%	27%	8%	18%	14%	16%	13%
OECD	25%	14%	26%	13%	23%	14%	23%	13%

Source: OECD (2013b)

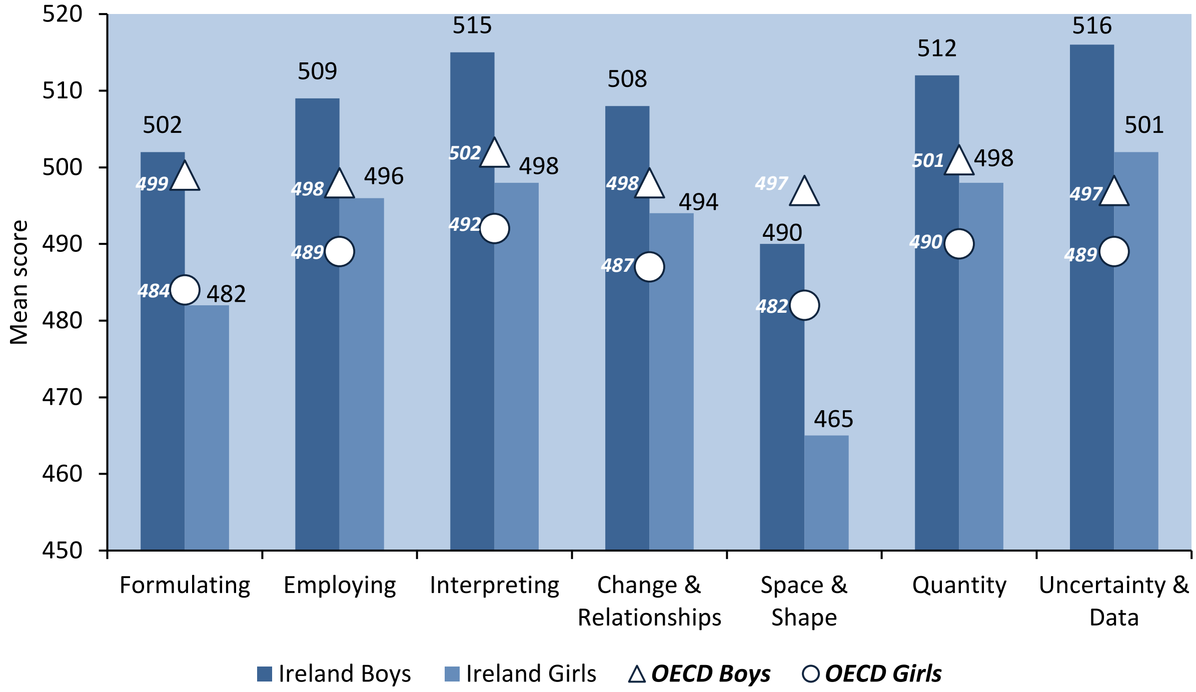
4.1.3. Gender Differences on Print Mathematics

Boys significantly outperform girls on the overall print mathematics scale in Ireland (509 compared to 494) and on average across OECD countries (499 compared to 489). The difference between boys and girls is slightly larger in Ireland (15 points) compared to the average difference across OECD countries (11 points), but not significantly so. Boys in Ireland score significantly higher than the OECD average for boys. Girls in Ireland also have a significantly higher mean score than girls across OECD countries.

In Ireland, boys significantly outperform girls on each of the mathematical process and content area subscales (Figure 4.3). Boys in Ireland have significantly higher mean scores than the average for boys across OECD countries on the Employing, Interpreting, Change & Relationship, Quantity and Uncertainty & Data subscales. The mean score for boys in Ireland does not differ from the corresponding OECD scores for the Formulating or Space & Shape subscales. Girls in Ireland have a significantly higher mean score on the Employing, Change & Relationships, Quantity and Uncertainty

& Data subscales than the average for girls across OECD countries, while the differences are not significant for the Formulating and the Interpreting subscales. Girls in Ireland perform significantly less well than the average for girls across OECD countries on the Space & Shape subscale.

Figure 4.3. Mean scores of boys and girls on the three mathematical process subscales and the four mathematical content subscales, in Ireland and on average across OECD countries



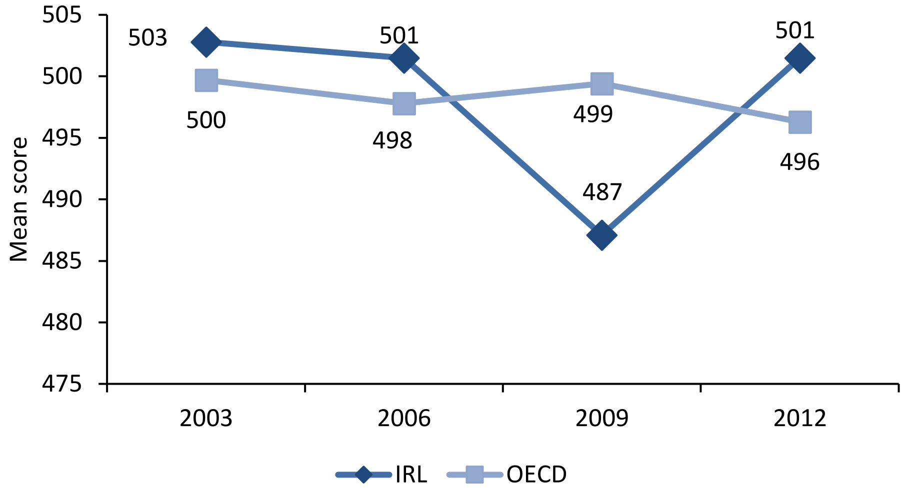
Source: OECD (2013b)

In Ireland, more girls than boys perform below Level 2 on the overall print mathematics scale (19% compared to 15%). The proportion of girls performing below Level 2 in Ireland is somewhat lower than the corresponding proportion across OECD countries (24%). Similarly, the percentage of boys in Ireland performing below Level 2 is considerably lower than the corresponding percentage across OECD countries (22%). On the other hand, in Ireland and on average across OECD countries, more boys than girls achieve print mathematics scores at Level 5 or above (13% compared to 9% for Ireland, and 15% compared to 11% across OECD countries). The percentages of boys and girls performing at or above Level 5 in Ireland are slightly below the corresponding OECD percentages.

4.1.4. Changes in Print Mathematics Performance over Time

In Ireland, mean print mathematics performance in 2012 is significantly higher than in 2009, but not significantly different to 2003 or 2006 (Figure 4.4). Ireland's performance is statistically significantly above the OECD average for the first time in 2012; however, this relates to a declining OECD average rather than a substantive improvement in performance in Ireland.

Figure 4.4. Mean scores on the overall mathematics scale for Ireland and the average across OECD countries,[4] 2003 to 2012



Source: OECD (2013b)

The percentage of students in Ireland performing below Level 2 is about the same in 2012 (17%) as in 2003 and 2006 (17% and 16%, respectively), but is lower than in 2009 (21%). The percentages of students in Ireland performing at Level 5 or above is also about the same in 2012 (11%) as in 2003 and 2006 (11% and 10%, respectively), but higher than in 2009 (7%). The mean scores of boys and girls on the overall print mathematics scale in Ireland in 2012 (509 and 494, respectively) are about the same as in 2003 (510 and 495, respectively) and 2006 (507 and 496, respectively), but are higher than in 2009 (491 and 483, respectively).

4.2. Performance on Computer-based Mathematics

Thirty-two countries/economies, including Ireland and 22 other OECD countries, also participated in the computer-based assessment of mathematics, which was administered for the first time in 2012. The mean computer-based mathematics score of students in Ireland (493) does not differ significantly from the corresponding OECD average (497; Table 4.5). Ireland’s score is ranked 15th among the 23 participating OECD countries and 20th among all 32 participating countries/ economies. When measurement and sampling error are accounted for (by applying a 95% confidence interval) the true rank for Ireland is between 12th and 18th among the 23 participating OECD countries and between 16th and 23rd among all 32 participating countries. Students in Ireland perform significantly less well on the computer-based assessment of mathematics than on the print mathematics assessment.

[4] Twenty-nine OECD countries have valid data for both 2003 and 2012. Therefore, for trend analysis, the OECD average score across cycles is computed as the average of these 29 countries.

Table 4.5. Mean country/economy scores for the computer-based mathematics scale and positions relative to the 23-country OECD mean score

Above OECD mean	Same as OECD mean	Below OECD mean
Singapore *(566)*	Italy *(499)*	Sweden *(490)*
Shanghai-China *(562)*	United States *(498)*	Russian Fed. *(489)*
Korea *(553)*	Norway *(498)*	Poland *(489)*
Hong Kong-China *(550)*	Slovak Republic *(497)*	Portugal *(489)*
Macao-China *(543)*	Denmark *(496)*	Slovenia *(487)*
Japan *(539)*	**Ireland *(493)***	Spain *(475)*
Chinese Taipei *(537)*		Hungary *(470)*
Canada *(523)*		Israel *(447)*
Estonia *(516)*		UAE *(434)*
Belgium *(512)*		Chile *(432)*
Germany *(509)*		Brazil *(421)*
France *(508)*		Colombia *(397)*
Australia *(508)*		
Austria *(507)*		

Source: OECD (2013b)

4.2.1. Performance of Higher- and Lower-achieving Students on Computer-based Mathematics

As with print mathematics, the performance of higher- and lower-achieving students can be observed by examining scores at the 10th and 90th percentiles. In Ireland, the score of students at the 10th percentile on the computer-based mathematics scale does not differ significantly from the corresponding 23-country OECD average (388 and 382, respectively). On the other hand, students performing at the 90th percentile on the computer-based mathematics scale in Ireland have a score of 594, which is significantly below the corresponding average of students across the 23 participating OECD countries (609). In Ireland, both higher- and lower- achieving students performed less well on the computer-based assessment of mathematics than on the print mathematics assessment, although the difference is greater among the higher-achieving students (-3 points for students at the 10th percentile and -16 points for students at the 90th percentile).

Student performance on computer-based mathematics can also be described in terms of proficiency levels (Table 4.2 provides a description of the types of skills that students at each proficiency level are capable of). In Ireland, 18% of students perform below Level 2, which is slightly lower than the 23-country OECD average proportion of 20%. The proportion of higher-achieving students (i.e., those at or above Level 5) on the computer-based mathematics scale in Ireland is 7%, which is somewhat lower than the corresponding 23-country OECD average proportion of 11%.

4.2.2. Gender Differences on Computer-based Mathematics

Boys significantly outperform girls on the computer-based mathematics scale in Ireland and on average across OECD countries (Figure 4.5). The difference between boys and girls for computer-based mathematics is somewhat larger in Ireland (19 points) compared to the average difference across OECD countries (13 points), but is not significantly different. The mean computer-based mathematics score for boys in Ireland does not differ significantly from the corresponding 23-country OECD average score. Girls in Ireland perform significantly less well on computer-based mathematics than girls across the 23 OECD countries. In Ireland, both boys and girls achieve higher mean scores on print mathematics than computer-based mathematics, although the difference is larger for girls (10 points) than for boys (7 points).

Figure 4.5. Mean scores of boys and girls on the computer-based mathematics scale, in Ireland and on average across the 23 participating OECD countries

Source: OECD (2013b)

4.3. Performance on Computer-based Problem Solving

Students in Ireland, along with students in forty-three other countries/economies, also participated in a computer-based assessment of problem solving, which was administered for the first time in 2012. Table 4.6 shows the average scores of all countries/economics that participated in the problem-solving assessment and their position relative to the average score of the 28 OECD countries that participated (500). In Ireland, students achieved a mean score of 498 on the assessment of problem solving, which does not differ significantly from the 28-country OECD average. Ireland's score is ranked 17th of the 28 participating OECD countries, and 22nd of all 44 participating countries/economies. When measurement and sampling error are accounted for (applying a 95% confidence interval), the true rank for Ireland is between 15th and 19th among the 28 participating OECD countries and between 20th and 24th among all participating countries/economies.

Table 4.6. Mean country/economy scores for the computer-based problem-solving scale and positions relative to the 28-country OECD mean score

Above OECD mean	Same as OECD mean	Below OECD mean
Singapore *(562)*	Austria *(506)*	Sweden *(491)*
Korea *(561)*	Norway *(503)*	Russian Fed. *(489)*
Japan *(552)*	**Ireland *(498)***	Slovak Republic *(483)*
Macao-China *(540)*	Denmark *(497)*	Poland *(481)*
Hong Kong-China *(540)*	Portugal *(494)*	Spain *(477)*
Shanghai-China *(536)*		Slovenia *(476)*
Chinese Taipei *(534)*		Serbia *(473)*
Canada *(526)*		Croatia *(466)*
Australia *(523)*		Hungary *(459)*
Finland *(523)*		Turkey *(454)*
United Kingdom *(517)*		Israel *(454)*
Estonia *(515)*		Chile *(448)*
France *(511)*		Cyprus *(445)*
Netherland *(511)*		Brazil *(428)*
Italy *(510)*		Malaysia *(422)*
Czech Republic *(509)*		UAE *(411)*
Germany *(509)*		Montenegro *(407)*
United States *(508)*		Uruguay *(403)*
Belgium *(508)*		Bulgaria *(402)*
		Colombia *(399)*

Source: OECD (2014)

4.3.1. Performance of Higher- and Lower-achieving Students on Problem Solving

In Ireland, the score of students at the 10th percentile is 378, which is similar to the corresponding 28-country OECD average score (375). The performance of students in Ireland scoring at the 90th percentile is also similar to the corresponding average across the 28 participating OECD countries (615 and 620, respectively).

As with mathematics, student performance in problem solving can also be described in terms of proficiency levels. Table 4.7 presents a description of the types of tasks that students at each of six proficiency levels are likely to succeed on. Level 6 is the highest level and students performing at this level are capable of successfully completing the most difficult PISA tasks. On the other hand, Level 2 is considered by the OECD as the baseline level of problem-solving proficiency at which students begin to demonstrate the skills that will allow them to participate effectively and productively in 21st century societies (OECD, 2014).

In Ireland, 20% of students are performing below Level 2 on the problem-solving scale, which is similar to the 28-country OECD average of 21%. Slightly fewer students in Ireland achieve scores at Level 5 or above compared to the 28-country OECD average (9% compared to 11%, respectively).

Table 4.7. Descriptions of the six levels of proficiency on the problem-solving scale

Level *(Cut-point)*	**Students at this level are capable of:**
6 *(683 and above)*	*Developing complete, coherent mental models of diverse problem scenarios, enabling them to solve complex problems efficiently*. They can explore a scenario in a highly strategic manner to understand all information pertaining to the problem. The information may be presented in different formats, requiring interpretation and integration of related parts. When confronted with very complex devices, such as home appliances that work in an unusual or unexpected manner, they quickly learn how to control the devices to achieve a goal in an optimal way. Level 6 problem-solvers can set up general hypotheses about a system and thoroughly test them. They can follow a premise through to a logical conclusion or recognise when there is not enough information available to reach one. In order to reach a solution, these highly proficient problem-solvers can create complex, flexible, multi-step plans that they continually monitor during execution. Where necessary, they modify their strategies, taking all constraints into account, both explicit and implicit.
5 *(618 to less than 683)*	*Systematically exploring a complex problem scenario to gain an understanding of how relevant information is structured*. When faced with unfamiliar, moderately complex devices, such as vending machines or home appliances, they respond quickly to feedback in order to control the device. In order to reach a solution, Level 5 problem-solvers think ahead to find the best strategy that addresses all the given constraints. They can immediately adjust their plans or backtrack when they detect unexpected difficulties or when they make mistakes that take them off course.
4 *(553 to less than 618)*	*Exploring a moderately complex problem scenario in a focused way.* They grasp the links among the components of the scenario that are required to solve the problem. They can control moderately complex digital devices, such as unfamiliar vending machines or home appliances, but they don't always do so efficiently. These students can plan a few steps ahead and monitor the progress of their plans. They are usually able to adjust these plans or reformulate a goal in light of feedback. They can systematically try out different possibilities and check whether multiple conditions have been satisfied. They can form a hypothesis about why a system is malfunctioning, and describe how to test it.
3 *(488 to less than 553)*	*Handling information presented in several different formats.* They can explore a problem scenario and infer simple relationships among its components. They can control simple digital devices, but have trouble with more complex devices. Problem-solvers at Level 3 can fully deal with one condition, for example, by generating several solutions and checking to see whether these satisfy the condition. When there are multiple conditions or inter-related features, they can hold one variable constant to see the effect of change on the other variables. They can devise and execute tests to confirm or refute a given hypothesis. They understand the need to plan ahead and monitor progress, and are able to try a different option if necessary.
2 *(423 to less than 488)*	*Exploring an unfamiliar problem scenario and understanding a small part of it*. They try, but only partially succeed, to understand and control digital devices with unfamiliar controls, such as home appliances and vending machines. Level 2 problem-solvers can test a simple hypothesis that is given to them and can solve a problem that has a single, specific constraint. They can plan and carry out one step at a time to achieve a sub-goal, and have some capacity to monitor overall progress towards a solution.
1 *(358 to less than 423)*	*Exploring a problem scenario only in a limited way, though often only when very similar situations have been encountered before.* Based on their observations of familiar scenarios, these students are able only to partially describe the behaviour of a simple, everyday device. In general, students at Level 1 can solve straightforward problems provided there is only a simple condition to be satisfied and there are only one or two steps to be performed to reach the goal. Level 1 students tend not to be able to plan ahead or set sub-goals.
Below Level 1 *(below 358)*	There were insufficient items to fully describe performance that falls below Level 1 on the problem-solving scale.

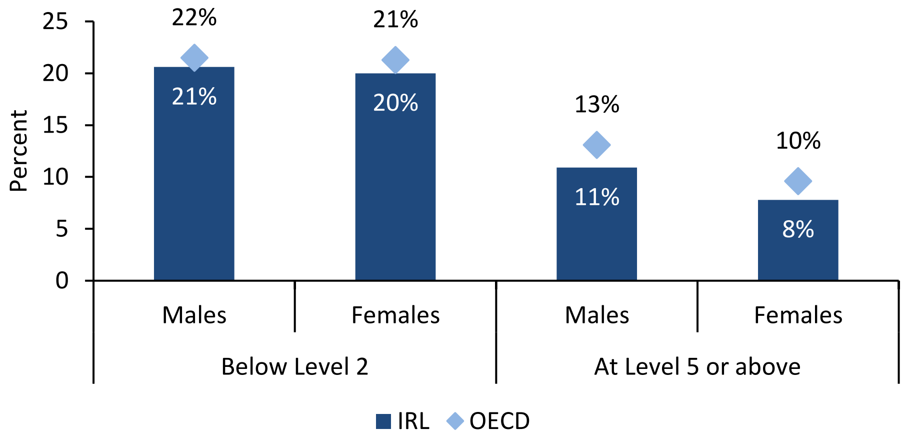
Source: OECD (2014)

4.3.2. Gender Differences on Problem Solving

In Ireland, boys have a slightly higher mean score than girls (501 compared to 496) on the problem-solving scale, but the difference is not statistically significant. Both boys and girls in Ireland have similar mean scores to the corresponding 28-country OECD average scores (503 for boys and 497 for girls). While the size of the gender difference in Ireland (5 points) is similar to the 28-country OECD average (7 points), the OECD average gender difference is statistically significant.

In Ireland, 21% of boys and 20% of girls perform below Level 2 on the problem-solving assessment (Figure 4.6). The corresponding 28-country OECD averages are only slightly higher. Slightly more boys (11%) than girls (8%) in Ireland are considered higher-achieving students (i.e. performing at Level 5 or above) on the problem-solving assessment. The proportions of higher achieving boys and girls in Ireland are marginally below the corresponding 28-country OECD averages.

Figure 4.6. Percentages of students performing below Level 2 and at Level 5 or above on the assessment of problem solving, in Ireland and on average across the 28 participating OECD countries



Source: OECD (2014)

4.3.3. Performance on Different Types of Problem-solving Items

In the PISA problem-solving framework, items are classified according to the nature of the problem and the cognitive processes involved in problem solving (see chapter 2 for an overview of the problem-solving framework). Separate subscales were not created for the nature of the problem or the cognitive processes; however the average percentage of correct responses to these items was computed.

With regards to the nature of the problem, in Ireland, the percentages of correct responses for static items and interactive items are almost identical (44% for static items and 45% for interactive items). Across the 28-particiating OECD countries 47% of static items were answered correctly compared to 44% of interactive items. Therefore, interactive items were found to be slightly harder than static items on average across the 28-participating OECD countries, indicating that performance on interactive items was marginally stronger than expected in Ireland.

Among the cognitive processes outlined in the problem-solving framework, a distinction is made between knowledge-acquisition (i.e. 'exploring and understanding' and 'representing and formulating' tasks) and knowledge utilisation tasks ('planning and executing' tasks). 'Monitoring and reflecting' tasks are considered to combine both knowledge-acquisition and knowledge-utilisation aspects and therefore are not included in this distinction. Relative strengths and weaknesses across the problem-solving processes can be identified for each country by comparing performance on the items measuring different problem-solving processes to the average performance of students across OECD countries (which is set at 1). The performance of students in Ireland on knowledge-acquisition tasks is in line with the 28-country OECD average performance, however students in Ireland are less likely to be successful on knowledge-utilisation tasks when compared to the OECD average (Table 4.8).

Table 4.8. Relative performance on the problem-solving knowledge acquisition and knowledge utilisation tasks, in Ireland and on average across OECD countries

	Knowledge acquisition		Knowledge utilisation
	Exploring and Understanding	Representing and Formulating	Planning and Executing
Ireland	1.06	0.97	0.91
OECD	1.00	1.00	1.00

Source: OECD (2014)

4.3.4. Performance on Problem Solving Compared to Performance on Mathematics, Reading and Science

Students in Ireland, and on average across the 28 participating OECD countries, perform less well than expected on problem solving, given their relatively strong performance in print mathematics, reading and science (-18 points for Ireland and -8 points for the 28-country OECD average).[5] In Ireland, 64% of students perform below the expected level on problem solving given their performance in print mathematics, reading and science, compared to 55% on average across the 28 participating OECD countries.

Summary

- Students in Ireland achieved a mean score of 501 on the print mathematics scale, which is significantly above the OECD average score of 494. Ireland's mean print mathematics score is ranked 13th out of 34 OECD countries and 20th out of all 65 participating countries. Ireland's true rank is between 11th and 17th among OECD countries and from 18th to 24th among all participating countries/economies, when sampling and measurement error are accounted for.
- The mean print mathematics score for Ireland in 2012 is significantly higher than in 2009, but does not differ from the mean score in 2003 or 2006.
- Seventeen percent of students in Ireland performed below proficiency level 2 on the print mathematics scale and 11% performed at Level 5 or above.
- Students in Ireland had significantly higher performance on the Employing and Interpreting process subscales than the corresponding OECD averages, but did not differ significantly on the Formulating subscale. Students in Ireland also had significantly higher performance on the Change & Relationship, Quantity and Uncertainty & Data

[5] Relative performance on problem solving is computed as the difference between actual performance and the fitted value from a regression using a second-degree polynomial as regression function (math, math sq., read, read sq., scie, scie sq., mathXread, mathXscie, readXscie) using data from students who participated in the computer-based assessments.

content subscales compared to the corresponding OECD averages, but performed significantly less well on the Space & Shape subscale.
- Boys significantly outperform girls on print mathematics in Ireland (509 compared to 494). The mean score for boys in Ireland is significantly higher than the OECD average for boys. Likewise, girls in Ireland also have a significantly higher mean score than the average for girls across OECD countries.
- The mean computer-based mathematics score for students in Ireland (493) does not differ from the average of the 23 participating OECD countries (497). Ireland's mean computer-based mathematics score is ranked 15th among 23 OECD countries and 20th among all 32 participating countries. Ireland's true rank is between 12th and 18th among the 23 participating OECD countries and from 16th to 23rd among all 32 participating countries/economies, when sampling and measurement error are accounted for.
- Students in Ireland performed significantly less well on the assessment of computer-based mathematics than on print mathematics.
- In Ireland, 18% of students performed below proficiency level 2 and 7% performed at Level 5 or above on the computer-based mathematics scale.
- In Ireland, the mean computer-based mathematics score for boys (502) is significantly higher than the mean score for girls (484).The mean score for boys in Ireland does not differ significantly from the OECD average for boys, while the mean score for girls in Ireland is significantly lower than the OECD average score for girls.
- The mean problem-solving score for Ireland is 498, which does not differ significantly from the OECD average score of 500. Ireland's mean problem-solving score is ranked 17th of 28 participating OECD countries and 22nd of all 44 participating countries. When measurement and sampling error are accounted for the true rank for Ireland is between 15th and 19th among OECD countries and between 20th and 24th among all participating countries/economies.
- In Ireland, 20% of students performed below proficiency level 2 and 9% performed at Level 5 or above on the problem-solving scale.
- In Ireland, boys and girls did not differ in terms of their problem-solving performance (501 and 496, respectively). Both boys and girls in Ireland have similar mean scores to the OECD average scores for boys (503) and girls (497).
- Students in Ireland performed better than expected on interactive problem-solving items. Compared to the OECD averages, students in Ireland are more likely to be successful on knowledge-acquisition tasks and less likely to be successful on knowledge-utilisation tasks.
- Students in Ireland perform less well than expected on overall problem solving, given their performance on the print assessments of mathematics, reading and science.

5. Factors Associated with Mathematics and Problem-solving Performance

This chapter considers a range of factors associated with performance on the PISA mathematics and problem solving tests. Three clusters of inter-related factors are considered: school-level factors, student-level factors and instructional factors. It should be noted that the majority of students who participated in PISA 2012 had no direct exposure to the new syllabus introduced under the Project Maths initiative.

5.1. School Factors

In Ireland, average school socio-economic status, as per PISA's index of economic, social and cultural status (ESCS)[6], is significantly correlated with print mathematics (r = 0.38), computer-based mathematics (r = 0.33) and computer-based problem solving (r = 0.32). These correlations indicate that as school socio-economic status increases, average performance on mathematics and problem solving also tends to increase. Conversely, on average, students in schools with lower socio-economic status tend to perform less well.

The difference in print mathematics performance between students attending socio-economically (ESCS) advantaged and disadvantaged schools (as defined by the OECD[7]) is 97 points. The corresponding average across OECD countries is 104 points, indicating a slightly lower association between school ESCS and student performance in Ireland. The correlation between school average ESCS and student ESCS in Ireland (r = 0.48) is in the moderate-to-strong range. This indicates that lower-ESCS students are likely to attend lower-ESCS schools and vice versa. PISA also suggests that, while high school-level ESCS can have a positive effect on the performance of lower-ESCS students, there can be a negative effect on the performance of high-ESCS students who attend lower-ESCS schools (Perkins et al., 2013, Figure 5.2).

Students in Ireland attending fee-paying schools had mean scores that were significantly higher than their counterparts in non-fee-paying schools. The difference for print mathematics was 57 points, while for computer-based mathematics, it was 38 score points. On problem solving, it was 56 points. Hence, there was a weaker association between school fee-paying status and mathematics when the outcome was computer-based mathematics, compared with print mathematics and computer-based problem solving.

In Ireland, 11.5% of students attended schools where students were grouped for mathematics instruction from the beginning of First Year. Just over three-quarters (77%) were grouped for mathematics from the beginning of Second Year, while 10% were grouped from the beginning of

[6] Socio-economic status in PISA is based on its index of Economic, Social and Cultural Status (ESCS), which is derived from six variables including parents' education, parents' occupations, cultural possessions, material possessions, home educational resources and the number of books available in the home.

[7] Advantaged schools are those where the typical student in the school is above the country average on the OECD's measure of socio-economic status, the economic, social and cultural status (ESCS) scale. This is a different approach to that used to identify school-level disadvantage in DEIS, where differences are 59.9 points in favour of students attending non-DEIS schools on paper-based mathematics, 51.2 on computer-based mathematics, and 49.8 on computer-based problem solving.

Third Year. About 2% of students were grouped at other times. Average performance on mathematics was significantly lower among students who were grouped from the beginning of First Year (467 points), compared with those who were grouped from the beginning of Second Year (507 points) or the beginning of Third Year (503 points).

5.2. Student Factors

5.2.1. Attitudinal Factors

Drawing on questionnaire items, PISA developed scales summarising students' mathematics self-efficacy (students' belief that they can solve various mathematical tasks), their mathematics self-concept (students' belief in how well they are doing in mathematics), and their anxiety about mathematics. Ireland's average scores on self-efficacy and self-concept were about the same as the corresponding OECD country averages, while the average level of anxiety was significantly higher. Correlations between these scales and performance on print mathematics in Ireland ranged from positive for self-efficacy (r = 0.55) and self-concept (r = 0.40) to negative for anxiety (r = -0.38). While boys had significantly higher average scores on self-efficacy and self-concept than girls, the opposite was found for anxiety, where girls had a significantly higher mean score. This is shown by the larger proportions of girls agreeing or strongly agreeing with statements such as 'I worry that I will get poor grades in mathematics' (69% girls, 55% boys), and 'I get nervous doing mathematics problems' (36% girls, 24% boys).

5.2.2. Motivational Factors

PISA constructed three scales describing aspects of motivation for mathematics. Each scale was based on several statements about which students had to indicate their level of agreement. The scales (examples of items) were: intrinsic motivation (e.g., I am interested in the things I learn in mathematics); instrumental motivation (e.g., Learning mathematics . . . will improve my career prospects); and perseverance with problem solving (e.g., When confronted with a problem, I do more than is expected of me). Students in Ireland achieved significantly higher average scores on all three scales, compared with students on average across OECD countries, with the greatest differences for instrumental motivation and perseverance. In Ireland, there was no difference in performance between boys and girls on intrinsic motivation, whereas boys had higher scores on instrumental motivation and perseverance. Correlations between motivational factors and performance were somewhat weaker than between attitudinal factors and performance. For example, the correlation between intrinsic motivation and print mathematics was 0.24, while that between perseverance and print mathematics was 0.26. The correlation between print mathematics and instrumental motivation was just 0.14. This indicates that, although average instrumental motivation was relatively high in Ireland, it was not strongly linked to mathematics performance.

5.2.3. Grade Level

As noted earlier, 15-year olds participating in PISA 2012 were drawn from a range of grade levels. Table 5.1 shows the distribution of performance across grade levels. For each domain, students in Second Year (fewer than 2% of 15-year olds) performed significantly less well on average than students in Third Year (61% of students), while students in Transition Year (one-quarter of students) performed significantly better. The gap in favour of students in Transition Year over students in Third Year was greatest for computer-based problem solving (32.5 points) and smallest for computer-based mathematics (21 points). Students in Transition Year performed significantly above their expected level in the computer-based assessment of problem solving, given their performance in print mathematics, reading and science (OECD 2014). Students in Fifth Year performed at about the same level as their counterparts in Third Year on print- and computer-based mathematics, and had a marginally higher, though not significantly different, mean score on problem solving. It should also be noted that year groups also vary with respect to their ESCS, with Second and Fifth years having lower average ESCS (-0.21 and -0.11, respectively) than Third years (0.13), and Transition Year students having the highest average level (0.27).

Table 5.1. Mean scores of students in Ireland at four grade levels on print mathematics, computer-based mathematics and computer-based problem solving

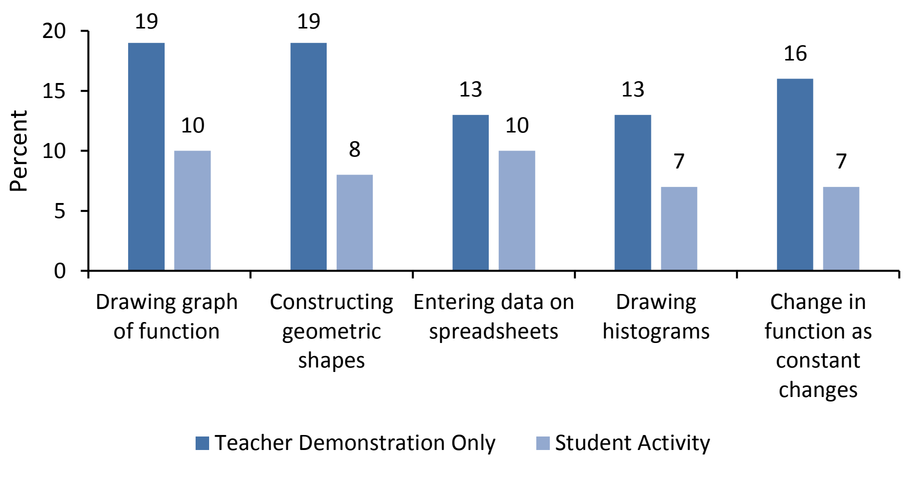
	% of students	Mean – print mathematics	Mean – computer-based mathematics	Mean – computer-based problem solving
Second Year	2	**445**	**458**	**443**
Third Year (Ref)	61	499	489	490
Transition Year	24	**523**	**510**	**523**
Fifth Year	13	502	487	498

Significant differences relative to the reference category (Third Year) shown in bold

5.2.4. Use of ICTs in Mathematics Classes

Students in Ireland in PISA 2012 tended not to use computers in mathematics lessons to any great extent, compared with their counterparts on average across OECD countries. The average score for students in Ireland on an index of computer use in mathematics (-0.15) was significantly below the OECD average (0.0), and considerably lower than countries such as Denmark (0.70), Norway (0.69) and Turkey (0.26). However, students in a number of countries with very high average achievement in mathematics also made relatively little use of computers in mathematics lessons, including Finland (average computer usage = -0.33), Korea (-0.36) and Japan (-0.62). Figure 5.1 shows that teacher demonstration of various mathematical procedures on computer occurred more frequently in Ireland for each of five procedures, compared to percentages of students implementing these procedures themselves. Overall, however, fewer than one-third of students reported that their teachers demonstrated the procedures or that they implemented them themselves.

Figure 5.1. Percentages of students in Ireland whose teachers only demonstrated various mathematical procedures using computers, and percentages of students implementing these procedures using computers



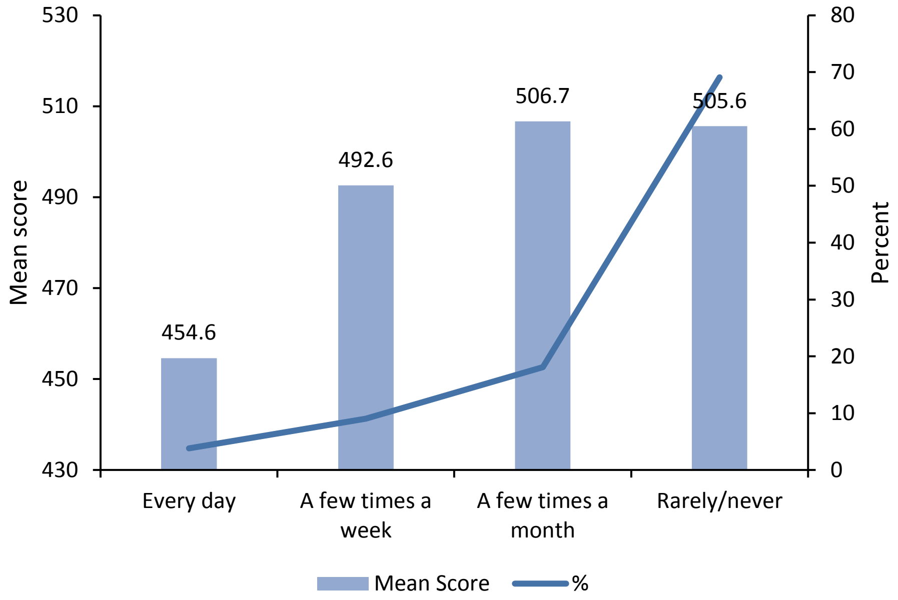
Use of ICTs in school in general by students in Ireland was greatest in Transition Year, while use of ICTs in mathematics lessons was greatest in Second Year, perhaps in an effort to address low mathematics achievement among 15-year olds at that grade level, i.e., students in Second Year in PISA may not be representative of Second Year students in general.

5.3. Instructional Factors

5.3.1. Teaching in Groups within Mathematics Classes

In Ireland, 69% of students reported that they never engaged in small-group activities in their mathematics classes. Just 4% worked in groups on a daily basis, 9% did so a few times a week, and 18% did so a few times a month. Figure 5.2 shows the average scores of students who worked in small groups with varying degrees of frequency. It indicates that regular engagement in small group work is more likely for lower-achieving than for higher-achieving students.

Figure 5.2. Percentages of students in Ireland working in groups in mathematics classes with varying degrees of frequency, and their mean mathematics scores



5.3.2. Opportunity to Learn

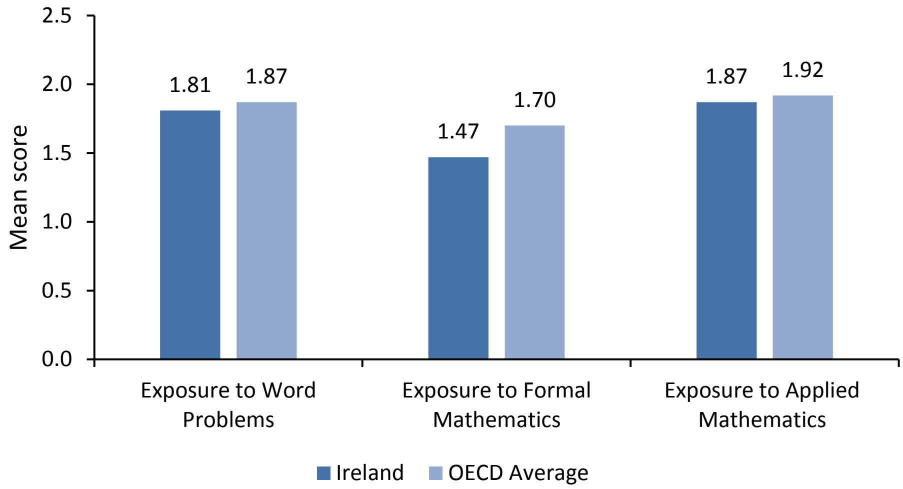
PISA 2012 examined how exposure to mathematical content (opportunity to learn) is associated with mathematics performance. Students were asked to indicate how often they had encountered various mathematical problems in school and how familiar they were with certain formal mathematics content. Three indices representing exposure to different types of mathematical problems were generated:

- Word problems (e.g., purchasing furniture at a discount; finding someone's age given his/her relationship to the age of others) (these problems are referred to in OECD (2014) as typical of those found in textbooks).
- Formal mathematics problems (e.g., solving 2x + 3 = 7; finding the volume of a box with sides of 3 cm, 4 cm and 5cm).

- Applied mathematics problems (e.g., using theorems to find the height of a pyramid; identifying if $(n + 1)^2$ can be a prime for any value of n; interpreting graphs; expressing relationships between variables such as heart rate and age).

Each scale ranged from 0 (no exposure) to 3 (high exposure). Figure 5.3 shows the averages scores of students in Ireland on each of these scales, and OECD average scores.

Figure 5.3. Mean scores on the indices of exposure to word problems, exposure to formal mathematics, and exposure to applied mathematics, in Ireland and on average across OECD countries

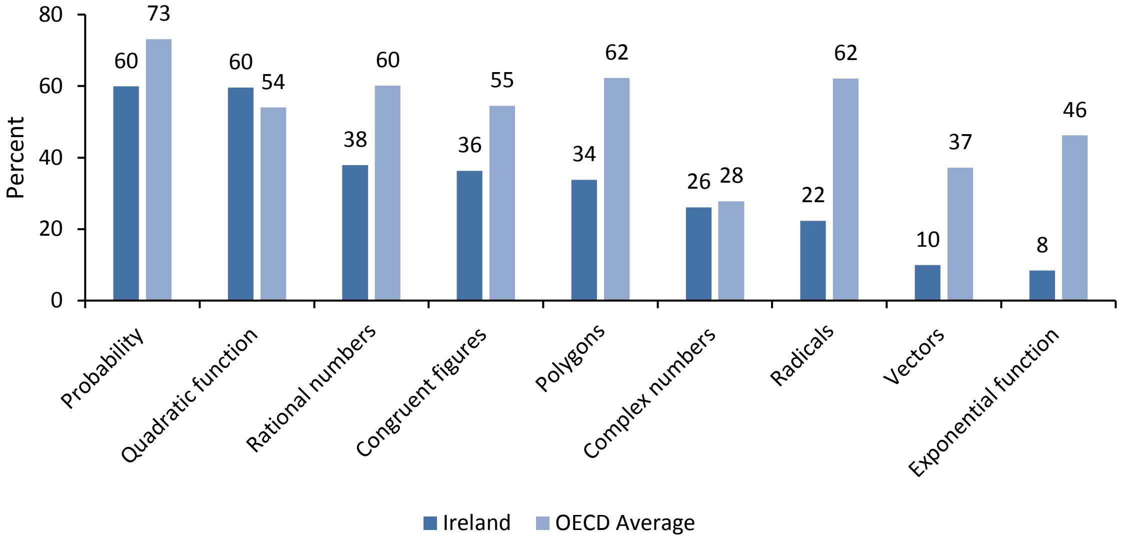


Ireland was close to, but marginally below, the OECD average score on the exposure to word problems and exposure to applied mathematics indices. However, just four OECD countries (Luxembourg, Switzerland, Iceland and Sweden) had lower average scores than Ireland on the index of formal mathematics problems. This is surprising given the strong focus in the pre-2010 Junior Certificate and Project Maths syllabi on aspects of formal mathematics, especially in the areas on algebra and geometry.

Higher scores on each of the indices were associated with higher performance on print mathematics at both school and student levels in Ireland and on average across OECD countries. However, while the relationship between exposure to word problems and print mathematics performance, and between formal mathematics and performance, was linear, it was curvi-linear (quadratic) for applied mathematics problems, indicating that, beyond a certain level, further exposure to applied problems was not associated with higher achievement. In relative terms, print mathematics performance was more strongly associated with exposure to formal mathematics (as defined by PISA) than exposure to word problems or exposure to applied mathematics problems.

PISA also asked students about their level of familiarity with each of several mathematical terms. Figure 5.4 indicates the percentages of students in Ireland and on average across OECD countries who reported that they ‘knew each term well and understood the underlying concept’ or had ‘heard it often’ (the other options were ‘heard it a few times’, ‘heard it once or twice’ and ‘never heard it’). However, it is possible that some students may have studied aspects of these topics without knowing the label for the aspect of mathematics represented.

Figure 5.4. Percentages of students who indicated that they knew each of several terms well or had heard them often, in Ireland and on average across OECD countries



While students in Ireland were very familiar with terms such as probability and quadratic functions, they lagged behind students on average across OECD countries on terms such as congruent figures, polygons, radicals, vectors and exponential functions.

Summary

- It should be noted that the data on which the current chapter is based were drawn from students prior to the full implementation of Project Maths at all class levels. Most students in the Third and Transition years in PISA 2012 had not studied directly under Project Maths at the time at which PISA 2012 was administered, while students in Fifth Year had studied the syllabus for just six months.
- School-level factors that correlated moderately or moderately to strongly with PISA print mathematics, computer-based mathematics and problem solving included socio-economic status, school disadvantaged status (as measured by PISA), and fee-paying status. PISA suggests that, while high school-level socio-economic status can have a positive effect on the performance of lower-level socio-economic status students, there can be a negative effect on the performance of higher-level socio-economic status students who attend lower-level socio-economic schools.
- Twelve percent of students were grouped by their schools for mathematics from the beginning of first year, and these students performed significantly less well than the 76% of students grouped from the beginning of second year, and the 10% grouped from the beginning of third year.
- A broad range of attitudinal and motivational factors correlated with students' performance on mathematics. Moderate to strong positive correlations were observed for self-efficacy in mathematics and mathematics self-concept, while a moderate negative correlation were observed for anxiety about mathematics. Boys in Ireland had higher average scores on self-efficacy and self-concept than girls, while girls had a higher anxiety score than boys. Correlations between motivational variables and print mathematics

were weaker than those between attitudinal variables and print mathematics. The correlation between instrumental motivation and performance was just 0.14.

- Students in Transition Year in Ireland had higher average performance on print mathematics, computer-based mathematics and computer-based problem solving, compared with their counterparts in Third and Fifth years, though this may be moderated by factors such as socio-economic status.
- Students in Ireland reported less usage of computers on average in mathematics classes than students on average across OECD countries, with students in Ireland lagging well behind their counterparts in countries such as Denmark and Norway. However, computer-usage was also low in a number of high-performing countries on PISA 2012 mathematics, including Korea and Japan. Students in Ireland, in classes in which computers were used in mathematics lessons, were more likely to observe their teachers demonstrating procedures such as drawing a graph of a function or entering data on spreadsheets than engaging in those activities themselves.
- While teachers may have little say on when students are grouped into mathematics classes, they can influence the extent to which students are grouped for mathematics activities during mathematics classes. However, group work within mathematics lessons was relatively infrequent, with a strong majority of students (almost 70%) rarely or never grouped for mathematics lessons.
- PISA developed a number of scales designed to evaluate students' opportunity to learn mathematics, based on reports provided by students (rather than analysis of curriculum documents or teacher reports). Average scores in Ireland on indices of exposure to word problems and exposure to applied mathematics were slightly below but not significantly different from the corresponding OECD average scores. However, students in Ireland had a lower mean score on exposure to formal mathematics. Higher scores on the indices were associated with high print mathematics performance at both school and student level, in Ireland and on average across OECD countries. However, the relationship between exposure to applied mathematics and print mathematics performance was curvi-linear, indicating that, beyond a certain point, further exposure to applied problems was not associated with increased performance. Furthermore, exposure to formal mathematics was more strongly associated with print mathematics performance than either exposure to word problems or applied mathematics problems.
- Students in Ireland also reported low familiarity with a number of mathematical concepts, compared with their counterparts on average across OECD countries. These included rational numbers, congruent figures, polygons, complex numbers, radicals, vectors and exponential functions. While lack of familiarity with some of these terms may be related to what is included and not included in the syllabus, it also raises issues about whether students in Ireland have the potential to learn more of these concepts at age 15, in line with students of the same age in other OECD countries.

6. PISA and Project Maths

As part of PISA 2012 in Ireland, a study was conducted to examine differences in performance in and dispositions towards mathematics and problem solving between students in 23 Initial Project Maths Schools[8] (where implementation of Project Maths began in 2008) and students in non-Initial schools (where it began in 2010). Whereas all students in the Initial schools had studied under Project Maths for a number of years prior to the implementation of PISA 2012, the vast majority of PISA students in non-Initial schools, including all Third-Year students, had no direct exposure to the Project Maths syllabus.

6.1. Performance on Print Mathematics

Students in Initial schools achieved a mean overall score of 505. Though higher than the mean score of students in non-Initial schools (501), it was not significantly different. The pattern of higher, but not significantly different, mean scores held for the three mathematics processes (Formulating, Employing and Interpreting) and the four mathematics content areas (Change & Relationships, Space & Shape, Quantity and Uncertainty & Data). In the case of Space & Shape, the mean score of students in Initial schools (486) was not significantly different from the OECD average (489), whereas for students in non-Initial schools (477), it was significantly lower. In line with the slightly higher overall mean score for students in Initial schools, marginally more students in Initial schools than in non-Initial schools scored at the highest level on the mathematics proficiency scale (2.2% compared to 1.7%), and marginally fewer students scored at the lowest levels (15.3% compared to 16.9%). The gap in favour of boys over girls in Initial schools on overall mathematics was 18 points, while in non-Initial schools, it was 15 points, with both differences reaching statistical significance. Boys in Initial schools had a significantly higher mean score on the Interpret process subscale than boys in non-Initial schools. There were no significant gender differences for mathematics content areas, though boys in Initial schools had a mean score (496) on Space & Shape that was higher than that of boys in non-Initial schools (490). Girls in Initial schools had a significantly higher mean score on Space & Shape (477) than their counterparts in non-Initial schools (465). Differences for girls were not statistically significant on any other content area or process subscale.

6.2. Performance on Computer-based Mathematics

The mean score of students in Initial schools on computer-based mathematics (497) was not significantly different from the mean score of students in non-Initial schools (493). While no significant differences were observed in the distribution of students by proficiency level across the two school types, the pattern of slightly fewer lower-achieving students, and slightly more higher-achieving students in Initial schools, was observed again. Within school categories, gender differences were statistically significant, with a larger gap in favour of boys in Initial schools (24 score points) compared with non-Initial schools (18).

[8] In 2008, there were 24 Initial Project Maths schools. Subsequently, one of these amalgamated with a non-Initial school, leaving 23 Initial Project Maths schools.

6.3. Performance on Computer-based Problem Solving

Students in Initial schools achieved a mean score of 502 on computer-based problem solving. This was higher than, but not significantly different from, the mean score of students in non-Initial schools (498). In Initial schools, girls had a mean score (502) that was higher than, but not significantly different from, the mean score of boys (501). In non-Initial schools, boys had a mean (501) that was higher than, but not significantly different from, the mean score of girls (495). There were no significant differences between students in Initial and non-Initial schools in the proportions scoring at each level on the problem-solving proficiency scales.

6.4. Links between Project Maths, pre-2010 Junior Certificate Syllabus and PISA

Three independent experts in second-level mathematics education undertook ratings of PISA 2012 secure items[9] on expected levels of student familiarity with each item (see Merriman et al., 2014 for additional details). Students studying Project Maths were judged to be more familiar, on average, with the concepts, contexts and processes underlying PISA items than students studying the pre-2010 syllabus. In particular, students studying Project Maths were judged to be more familiar with Space & Shape items, compared with the pre-2010 syllabus, where, perhaps, spatial mathematics was not emphasised to the same degree as more theoretical aspects of geometry. Raters also pointed to the high level of basic literacy required to successfully complete PISA 2012 mathematics. PISA mathematics was judged to neither encompass everything in mathematics nor everything in the Irish curriculum. PISA was described by our expert raters as linear, with little ambiguity and few opportunities for alternative approaches or lateral reasoning.

6.5. Dispositions Towards Mathematics

Drawing on questionnaire data from PISA, students in Initial and non-Initial schools were compared on a number of attitudinal and behavioural measures. Statistically significant differences were observed on seven scales. Students in Initial schools had higher mean scores on mathematics anxiety (indicating a higher average level of anxiety) and on self-responsibility for failure in mathematics (indicating that they were more likely to attribute failure in mathematics to their own inability or lack of effort rather than to factors such as their teacher, course materials or luck). Students in non-Initial schools had significantly higher mean score on mathematics self-concept and on mathematics related behaviours (such as engagement in mathematics clubs or competitions), mathematics-related intentions (such as a preference for mathematics-related careers or courses in the future), and mathematics-related subjective norms (such as a perception that parents and friends had more positive attitudes towards mathematics). Scales on which differences between students in Initial and non-Initial schools were not statistically significant include mathematics self-efficacy, instrumental motivation, and perseverance. The higher average levels of mathematical anxiety reported by students in Initial schools may have arisen because students in Initial schools would have been the first to sit state examinations that included items based on the Project Maths syllabus.

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[9] These are items that may be used to assess mathematics in future cycles of PISA, and so were not released by the OECD following PISA 2012.

Summary

- There were no significant differences in the performance of students in Ireland in Initial Project Maths schools or in non-Initial schools in PISA 2012 on overall print mathematics, computer-based mathematics and computer-based problem solving, though, in each case, students in Initial schools had slightly higher mean scores than their counterparts in non-Initial schools. A similar pattern held for the three mathematics processes and four mathematical content areas assessed on print mathematics. However, in the case of Space & Shape, the mean score of students in Initial schools (486) was not significantly different from the corresponding OECD average (489), whereas the mean of students in non-initial students (477) was significantly lower.
- Boys in both Initial and non-Initial schools had significantly higher mean scores than girls on both overall print mathematics (with differences of 18 points and 15 points respectively) and computer-based mathematics (24 and 20 points). On computer-based problem solving, girls in Initial schools and boys in non-Initial schools had higher, but not significantly different, mean scores.
- An analysis by experts in mathematics education indicated that students studying the Project Maths syllabus were more likely to be familiar with the concepts, contexts and processes underpinning PISA items, compared with their counterparts in non-Initial schools who mainly studied under the pre-2010 Junior Cycle mathematics syllabus. In particular, students studying Project Maths were judged to be more familiar with Space & Shape items, compared with students studying the older syllabus.
- Students in Initial Project Maths schools had a significantly higher mean score than their counterparts in non-Initial schools on the PISA mathematics anxiety scale. This may have arisen because, in addition to being among the first students to study under the Project Maths syllabus, students in Initial schools were among the first to sit the Junior Certificate mathematics examination under the Project Maths syllabus.
- The findings reported in this chapter are broadly similar to those reported in formal evaluations of Project Maths (Jeffes et al., 2012, 2013). For example, Jeffes et al. (2013) observed few overall differences in performance between students studying the Project Maths syllabi in Initial and non-Initial schools on released items drawn from the Trends in International Mathematics and Science (TIMSS) study. Noting that traditional approaches to instruction continued to be widespread, even after the implementation of Project Maths in all schools, they called for 'more high quality tasks that require [students] to engage with the processes promoted by [Project Maths], including: problem solving; drawing out connections between mathematics topics; communicating more effectively in written form; and justifying and providing evidence for their answers (p. 5).

7. Engagement with PISA Mathematics Items

Results for Ireland showed significant improvements in mathematics achievement between 2009 and 2012. One possible explanation for this improvement is that students were more likely to engage with and to attempt test items in 2012 compared to 2009. This chapter presents the results of analyses examining changes in students' engagement with test items across PISA cycles. Also presented in this chapter are the results of a nationally-developed questionnaire, administered as part of PISA 2012, which asked students about their test-taking behaviour.

7.1. Engagement with Mathematics Items

PISA 2012 mathematics data were compared with data from previous cycles to explore patterns of non-response as students progressed throughout the test. Items that were selected for analysis were common across three cycles of PISA (2006, 2009, and 2012).

Table 7.1 presents the average percentage of selected mathematics items answered correctly, incorrectly, missing or not reached for quartiles (booklet quarters) 1 and 4, in 2006, 2009 and 2012. The percentages of items answered correctly in quartile 1 decreased slightly between 2006 and 2009 (from 47% to 44%) and subsequently increased in 2012 (47%). In each cycle, the percentage of items answered correctly was lower in quartile 4 than in quartile 1; however, the decrease between quartiles was greater in 2009 (-5%) than in 2006 and 2012 (-3%). The percentage of items answered incorrectly was similar across all cycles in quartile 1. However, there was a slight decrease in the percentage of items answered incorrectly between quartiles 1 and 4 and that decrease was greater in 2009 (-4%) than in 2006 (-1%) and 2012 (-2%).

The percentage of items that were missed or not reached in quartile 1 was somewhat greater in 2009 (10%) than in 2006 (8%) and 2012 (7%), suggesting somewhat higher levels of disengagement among students in 2009, even from the beginning of the assessment. Also, there was an increase in the percentage of missed or not reached items between quartiles 1 and 4 across all cycles, but this increase was considerably greater in 2009 (+9%) compared to 2006 and 2012 (+4%). Interestingly, almost 20% of items were either missing or not reached in quartile 4 in 2009, compared to about 12% in 2006 and 2012.

These findings indicate that students were slightly less well able to answer the items in 2009 than in 2006 or 2012 when they were presented at the beginning of the booklet. However, the larger drop off in the percentage of items answered correctly along with the considerable increase in the percentage of items that were missed or not reached in 2009 compared to the other cycles suggests that students were less engaged with the assessment in 2009 compared to 2006 and 2012. Although it is difficult to quantify the amount, it is likely that at least part of the drop in mathematics achievement observed in Ireland in 2009 was due to lower levels of engagement with assessment among participating students.

Table 7.1. Average percent correct, incorrect, and missing or not reached for students in Ireland for mathematics in quartiles 1 and 4; 2006, 2009 and 2012

		Quartile 1	Quartile 4	Difference (Q4-Q1)
% Correct				
	2006	47%	44%	-3
	2009	44%	39%	-5
	2012	47%	44%	-3
% Incorrect				
	2006	45%	44%	-1
	2009	45%	41%	-4
	2012	46%	44%	-2
% Missing or not reached				
	2006	8%	12%	4
	2009	10%	20%	10
	2012	7%	12%	5

Data in this table are based on mathematics block M2/M1

7.2. Students' Test-taking Behaviour

A nationally developed questionnaire which asked questions about students' test-taking behaviour was administered directly after both the print and computer-based assessments in Ireland in 2012. The questionnaire was designed to gather information about the strategies students used for answering questions they did not know the answer to (multiple-choice and written response), if they had skipped questions they felt they had a good chance of getting right and why, how easy or difficult they had found the test, their level of interest in it, their level of concentration during the test, and how long it took them to complete it. The questions asked in the test-taking behaviour questionnaire referred to all test items in the PISA assessments and therefore include reading and science as well as mathematics and problem-solving items. Of the 5,016 students who completed the print assessment, 4,949 also completed the test-taking behaviour questionnaire, while 2,391 completed a test-taking behaviour questionnaire after completing the computer-based assessment.

Table 7.2 presents the percentage of students who indicated various strategies used if they did not know the answer to a multiple-choice or written response question on the print assessment and their associated average print mathematics score. Students who reported that they kept working on multiple-choice (29%) and written response (25%) questions until they thought they had the right answer obtained the highest average print mathematics scores, while those who indicated that they skipped questions and did not go back to them had the lowest average scores. Significantly more boys (31%) than girls (27%) reported that they kept working on multiple-choice questions until they thought they had the right answer. On the other hand, significantly more girls (27%) than boys (22%) reported guessing the answer to written response questions.

Table 7.2. Percent of students who indicated which strategy they used if they did not know the answer to multiple-choice and written response questions on the print assessment

What did you usually do if you did not know the answer to a….	Multiple-choice question		Written response question	
	%	Mean score	%	Mean score
I kept working on it until I thought I had the right answer	29%	541	25%	537
I skipped it and went back to it later	20%	498	26%	506
I guessed the answer	30%	478	24%	483
I skipped it and did not go back to it	3%	407	11%	458
I did a mixture of these things	18%	500	14%	502

A similar pattern was observed among students who sat the computer-based assessment. Those who indicated that they kept working on multiple-choice questions (34%), typed-response questions (38%) and questions that required students to perform an action (46%) until they had the right answer obtained the highest computer-based mathematics scores (Table 7.3). On the other hand, students who indicated that they skipped questions that they did not know the answer to had the lowest computer-based mathematics scores, across all question types.

Table 7.3. Percent* of students who indicated which strategy they used if they did not know the answer to multiple-choice questions, questions where they had to type the answer and questions where they had to perform an action on the computer-based assessment

What did you usually do if you did not know the answer to a….	Multiple-choice question		Question where you type the answer		Question where you perform an action	
	%	Mean score	%	Mean score	%	Mean score
I kept working on it until I thought I had the right answer	34%	522	38%	521	46%	510
I guessed the answer	49%	484	40%	481	36%	483
I skipped it	8%	440	10%	458	6%	457
I did a mixture of these things	10%	502	12%	495	12%	492

*Due to rounding values may not add to exactly 100%

Just over 15% of students said they skipped questions on the print assessment that they felt they had a good chance of getting right; 28% did so because they ran out of time, 23% because they felt it was not worth the effort, 26% did so for a mixture of these reasons and 23% for another reason. Boys (17%) were more likely than girls (13%) to skip questions they felt they had a good chance of getting right. Boys were also more likely than girls to skip these questions because they felt it was not worth the effort (27% and 18%, respectively), while girls were slightly more likely than boys to skip questions because they ran out of time (30% and 27%, respectively). Students who skipped items they felt they had a good chance of getting right because they felt it was not worth the effort did not answer an average of 24% of mathematics items, while those who skipped items because they ran out of time did not answer an average of 13% of mathematics items.

The percentage of students who skipped questions that they felt they had a good chance of getting right was higher for the computer-based assessment, at 22%. The reasons for skipping such questions also differed slightly; 38% did so because they ran out of time, 22% felt it was not worth the effort and the remaining 40% did so either for a mixture of these reasons or for another reason. As with the print assessment, boys were more likely than girls to skip computer-based questions that they felt they had a chance of getting right (24% and 20%, respectively). A quarter of boys did so because they felt it was not worth the effort, compared to 17% of girls. On the other hand, 40% of boys and 37% of girls skipped these questions because they ran out of time.

About 22% of students rated the PISA print test as easy, while 15% rated it as difficult or very difficult. Most students indicated that they were very (14%) or quite interested (46%) in the print test; however, a substantial minority were not very interested (32%) or not interested at all (8%). Boys (28%) were more likely than girls (15%) to rate the print test as easy, but did not differ in terms of their interest level (15% of boys and 14% of girls said they were very interested in the print assessment). Students who reported being less interested in the print test were more likely to skip items and students were more likely to skip mathematics items than reading or science items, regardless of their interest level in the print assessment (Table 7.4).

Table 7.4. Average percent of items that were missed or not reached for students in Ireland for print mathematics, reading and science in 2012 by interest level in the print assessment

	% of items missed or not reached		
	Mathematics	Reading	Science
Very interested	5.1	2.7	4.4
Quite interested	7.4	3.6	5.4
Not very interested	10.8	6.3	6.1
Not interested at all	15.5	11.0	9.9

Twenty-nine percent of students rated the computer-based test as easy, while 22% thought it was difficult or very difficult. The majority of students (63%) indicated that they found the test very or quite interesting. Boys were more likely than girls to rate the test as easy (31% compared to 26%) but were about equally as likely to indicate that they found the test very or quite interesting (64% compared to 63%).

The findings from the test-taking behaviour questionnaires highlight the relationship between students' interaction with the tests and their achievement scores and are particularly important in the context of the growing use of assessments in schools. Also, differences between boys and girls in their perception of the PISA assessments and the strategies used for answering different question types indicate the need for different approaches to increasing engagement among different groups of students.

Summary

- A larger drop in the percentage of items answered correctly along with a considerable increase in the percentage of items that were missed or not reached in PISA 2009 compared to other cycles, suggests that students in Ireland were less engaged with the assessment in 2009 compared to 2006 and 2012.
- In 2012, students who reported that they kept on working on questions until they thought they had the correct answer obtained the highest average mathematics score, while those who skipped questions that they did not know the answer to had the lowest average score.
- In 2012, 15% of students skipped questions on the print assessment that they felt they had a good chance of getting right, while 22% of students did so on the computer-based assessment. Boys were more likely than girls to skip questions on the print assessment that they thought they had a good chance of getting right.
- About 22% of students rated the print assessment as easy, while 29% of students rated the computer-based assessment as easy. About 60% of students said they found the print and computer-based assessments very or quite interesting. Boys were more likely than girls to rate the assessments as easy, but they did not differ in terms of their interest level.
- Students who reported being less interested in the print assessment were more likely to skip items and students were more likely to skip mathematics items than reading or science items.

8. What does it Mean for Teachers?

This chapter considers the main findings from PISA 2012 and their implications for teaching and learning. Four themes are examined: performance on the Space & Shape content area, the relative underperformance of higher-achieving students in mathematics, the relationship between expectations and performance, and alleviating anxiety about mathematics. It should be noted that the vast majority of students who participated in PISA 2012 did not have any formal exposure to the new Project Maths curriculum at the time of assessment and the conclusions here should be considered in that context.

8.1. A Topic of Concern: Space & Shape

While students in Ireland in PISA 2012 achieved mean scores that were above the corresponding OECD average scores on the Change & Relationships and Uncertainty content areas, and performed at about the OECD average on Quantity, they performed at a level that was below the OECD average on Space & Shape, with girls in Ireland doing particularly poorly. Students in Ireland performed below the OECD average on Space & Shape in PISA 2003, when mathematics was also a major assessment domain in PISA. Students in the 23 Initial Project Maths schools who participated in PISA 2012 in Ireland achieved a mean score on Space & Shape that was higher than, though not significantly different from, the mean score of students in non-Initial schools. Boys in Initial schools performed at a level that was not significantly different from the OECD average for boys, whereas girls performed at a level that was significantly below the OECD average for girls.

Ireland is not unique in performing relatively poorly on Space & Shape. In 2012, students in the United Kingdom, the United States and New Zealand achieved mean scores on Space & Shape that were well below the OECD average. In all of these countries, male students outperformed females, though the difference was considerably larger in Ireland and New Zealand than in the United States and the United Kingdom. Students in a number of European countries, including Austria, Belgium, Estonia, Germany, the Netherlands and Poland performed at a level that was significantly above the OECD average, though none performed as well on Space & Shape as Asian countries such as Chinese-Taipei, Japan and Korea. In general, countries with above average scores on Space & Shape performed well on overall PISA mathematics.

Items on PISA Space & Shape span a range of activities such as understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. Geometry is viewed by PISA as being central to Space & Shape, though aspects of other content areas such as spatial visualisation, measurement, number and algebra are also drawn on. The manipulation and interpretation of shapes in settings such as dynamic geometry software and Global Position System (GPS) tools are included in the domain, though not necessarily represented in current PISA Space & Shape items.

It is unclear why students in Ireland in general, and girls in particular, struggle with Space & Shape. One reason may relate to curriculum content. For example, Close (2006) pointed to a significant mismatch between PISA Space & Shape and geometry on the pre-2010 Junior Certificate syllabus (under which most PISA 2012 students in Ireland studied). He was unable to classify any Junior Certificate geometry items as being PISA Space & Shape type items, though some Space & Shape items could be classified as belonging to other Junior Certificate content areas such as Applied Arithmetic & Measure.

Even though space and shape may not be prominent in Junior Certificate mathematics, it is likely that at least some elements of space and shape that are not covered are important, and that students would benefit from interacting with these elements, not only to improve their understanding of mathematics in general, but to enable them to better understand science and related areas (Wai, Lubinski & Benbow, 2009).

There might also be value in providing direct instruction in visual-spatial skills ('spatially-enriched education') to students. Greater use of software linked to spatial reasoning, geometry and functions by both teachers and students could help with visualisation skills, including manipulation of moving shapes. Indeed, Uttal et al. (2013) have shown that computer programmes can be effective in developing students' spatial reasoning skills, and improving their performance in both mathematics and science. There might also be a benefit to many students if teachers were to adopt more visual approaches to teaching fundamental mathematics concepts (Sobanski, 2002).

More detailed input on space and shape could be provided in the context of general mathematics classes, Transition Year mathematics modules, and short courses with an emphasis on mathematics. Opportunities for integrating concepts associated with the Space & Shape content area could also be availed of in subjects like geography (e.g., location of cities in relation to one another, map reading, orientation, grid references, latitude and longitude, time zones), history (location, buildings, archaeology, sense of time and space), science (shape in natural phenomena, properties of particles), and literature (timescales, direction) (see Fox & Surtees, 2010). In devising interventions, particular attention needs to be given to addressing girls' development of spatial concepts and spatial reasoning.

8.2. Underperformance of High-achieving Students

While the overall mathematics and problem-solving performance of students in Ireland is at average or slightly above average levels, there is evidence that higher-achieving students in Ireland are underperforming relative to their counterparts in other countries. For example, despite Ireland's higher than average performance on print mathematics, the proportion of students in Ireland who are able to answer the most difficult PISA questions is significantly below the corresponding OECD average and the score of students at the 90th percentile does not differ significantly from the corresponding OECD average score. Similarly, while overall performance for computer-based mathematics is at average levels, the performance of higher-achieving students in Ireland on the computer-based assessment of mathematics is significantly below the corresponding OECD average score, and the proportion scoring at the highest proficiency levels is also lower than on average across OECD countries. These findings indicate a greater need to cater for more able students.

Particular areas in which higher-achieving students are underperforming include the Change & Relationships and Space & Shape mathematical content areas, and the Formulating mathematical process. There is also evidence that students in Ireland are underperforming in areas of problem solving that require planning and executing. Students could benefit from engagement in higher-level tasks in these areas, such as engaging with problems in novel contexts and using technology to explore different solutions to problems, as well as exploring mathematical thinking more generally in a metacognitive (reflective) way. Initiatives such as Maths Circles, "Maths Eyes" and the Khan

Academy could offer opportunities for students to engage in problems in new ways and to encourage self-directed learning among higher-performing students. Similarly, interactive platforms such as GeoGebra could offer students opportunities to engage in Space & Shape related tasks, such as rotation, which may broaden their understanding of the current curriculum. Transition Year could also offer an opportunity for higher-achieving students to explore new approaches to some of the problems they would have encountered as part of the junior cycle curriculum and to increase engagement with mathematics through initiatives such as LearnStorm (http://www.learnstorm2016.org).

Strategies which may assist students, and which students in Ireland were asked to perform less often than their OECD counterparts, include using their own procedures for solving complex problems and solving problems which could be solved in different ways.

Such activities could be supported by prioritising a more dialogical pedagogy in mathematics classrooms where students are afforded more opportunities to talk about their mathematical thinking in a metacognitive (reflective) way to support their ability to solve mathematical problems. Consistent with Project Maths, such dialogue could be promoted in small groups contexts (where a group works together to solve one or more complex problems) and in whole-class settings.

8.3. Raising Expectations for Success

Notwithstanding the relatively strong performance of lower-achieving students in Ireland on PISA 2012 mathematics, and an overall mean score that was significantly above the OECD average, a number of factors lead to the conclusion that students in Ireland are, in general, capable of performing at a higher level.

It is noteworthy that students in Ireland perform less well on PISA mathematics than on PISA reading literacy and science. In PISA 2012, while students in Ireland achieved a mean score that was just above the OECD average on mathematics, they performed well above the OECD average on reading literacy and science. Indeed, only Japan and Korea among OECD countries achieved a higher mean score than Ireland on reading literacy. Given that countries in PISA that perform well in one domain usually perform well on the other domains, one wonders why students in Ireland perform less well in mathematics, and whether the new mathematics syllabus on its own is capable of bridging the gap and raising performance to the levels observed in reading literacy and science.

The data on opportunity to learn presented in Chapter 5 indicate that students in Ireland report significantly less exposure to formal mathematics, compared with average exposure across OECD countries (though exposure to word problems, and problems in applied contexts were similar to OECD average levels). Furthermore, fewer students in Ireland than on average across OECD countries reported that they understood well or had often heard terms such as ‘rational numbers’, ‘congruent figures’, ‘exponential functions’ or ‘polygons’. While it may be that students in Ireland encounter significant additional formal mathematics in contexts other than those presented to them on the PISA student questionnaire, it also seems likely that students in Ireland are capable of embracing a broader range of formal mathematical concepts. Indeed, proponents of opportunity to learn in mathematics education (e.g., Schmidt et al., 2015)[10] argue that inequalities in mathematics

[10] Schmidt et al.’s study shows small gaps in opportunity to learn formal mathematics and average mathematics performance between the least and most disadvantaged schools relative to the corresponding OECD averages, but large gaps on average within schools on both measures.

education (where students of lower socio-economic status are provided with fewer opportunities to encounter more challenging content) could be addressed if schools and teachers held the view that all students should be exposed to challenging mathematics content. According to Boaler and Forster (2014), this might be achieved by 'de-tracking' classes (that is, creating more heterogeneous mathematics classes within schools), and by:

- Encouraging many approaches to solving mathematical problems rather than a single, pre-defined approach
- Embracing mistakes and treating them as rich learning opportunities by providing feedback that draws on and extends students' thought processes, and only then offering suggestions for corrections and improvement
- Holding the expectation that all students can achieve high levels of performance in mathematics
- Focusing on processes as well as content, and using discussion (rather than right/wrong) as a basis for developing mathematical thinking

8.4. Overcoming Mathematical Anxiety

The mean score for students in Ireland on the PISA anxiety about mathematics scale was higher than on average across OECD countries, indicating greater levels of anxiety in Ireland. Moreover, girls in Ireland had significantly higher levels of anxiety than boys, while students in Initial Project Maths schools had higher anxiety than students in non-Initial schools. The relatively strong negative correlation between anxiety and performance in Ireland (-.33) does not necessarily indicate a causal relationship. Indeed, according to Carey, Hill, Devine and Szücs (2016), there is evidence of a bi-directional relationship between mathematical anxiety and mathematics performance, in which anxiety and performance influence one another in a 'vicious cycle'. Tobias (1993) notes that mathematical anxiety can occur at all levels (primary, post-primary and higher education) and can be high among parents as well as students.

The role schools and teachers play in reducing anxiety about mathematics should be examined. The alleviation of anxiety about mathematics is a significant challenge, given the range of factors that may lead to anxiety, including parents' and society's attitudes towards mathematics, students' perceptions of their own mathematical ability, and their concerns about performance on mathematics tests and examinations. Nevertheless, many of the hands-on activities underpinning Project Maths as well as changes to the Junior Certificate examination should lead to learning environments in which teachers' expectations for higher performance levels can co-exist with reduced levels of anxiety among students. At a minimum, these environments should include high levels of student success combined with appropriate process-based feedback when students make errors, as well as opportunities for students to explore, conjecture and think rather than rote learn. It is also important for students to see mathematics being used in positive and relevant ways (Tobias, 1993) and to develop 'growth mindsets', whereby students believe that ability is not fixed, and that they can achieve higher levels of performance, engagement and persistence (Dweck, 2006). In particular, there should be an emphasis on raising achievement and mathematical self-concept of girls, with a view to ensuring that their anxiety levels are significantly reduced.

Summary

- A number of interlinked issues in mathematics education that arise from an analysis of the performance and attitudes of students in Ireland in PISA 2012 were identified, including low average performance on the Space & Shape content area, especially among girls, underperformance among higher-achieving students, limited expectations for success (as evidenced by limited exposure to aspects of formal mathematics), and high average levels of anxiety about mathematics, again with particular reference to girls. In considering how these issues might be addressed by schools and teachers, it should be noted that most 15-year olds in Ireland who participated in PISA 2012 had not experienced the Project Maths syllabus, though, among those who had, average performance was marginally, but not significantly, higher, while the average level of anxiety about mathematics was significantly higher.
- A key finding in PISA 2012, as well as in PISA 2003, was the low average level of performance by students in Ireland on the Space & Shape mathematics content area. Indeed, it would be difficult to raise average overall performance on PISA mathematics in Ireland without also addressing low performance on Space & Shape, especially among girls. A number of approaches to addressing low performance on Space & Shape were proposed, including the provision of computer-based interventions, the provision of short courses on spatial reasoning, and the stronger integration of concepts associated with this content area into subjects such as history, geography and science.
- Another key finding in 2012, and in 2003, concerned the relatively low performance of higher-achieving students in Ireland. Greater use of relevant technology by such students in mathematics classes was identified as being potentially helpful, as was greater use of more open-ended problems, including problems that could be solved in multiple ways. Greater use of dialogical pedagogy in mathematics classrooms where students are afforded more opportunities to talk about their mathematical thinking in a metacognitive (reflective) way was also proposed.
- The finding that students in Ireland had fewer opportunities to engage in formal mathematics than students on average across OECD countries was identified as a matter of concern, and linked to a need to adopt higher expectations for success. A number of strategies designed to support students in their encounters with more complex mathematics (some of which are consistent with Project Maths) were identified including embracing student errors and treating them as valuable learning opportunities, by providing feedback that extends students' thinking, and holding the expectation that all students can achieve high levels of performance in mathematics.
- Finally, the high average level of anxiety about mathematics among students in Ireland, and especially among girls, was noted, and it was acknowledged that the relationship between mathematics achievement and anxiety may be bi-directional. Suggestions for alleviating students' anxiety about mathematics included ensuring that students reach high levels of success, providing processed-based feedback when students make errors, and making opportunities for students to explore, conjecture and think rather than rote learn. The importance of encouraging positive growth mindsets in students that enable higher levels of performance, engagement and persistence was noted.

References

Boaler, J., & Foster, D. (2014). *Raising expectations and achievement: The impact of wide-scale mathematics reform giving all students access to high quality mathematics*. Unpublished paper, accessed at http://www.youcubed.org

Carey, E., Hill, F., Devine, A., & Szücs, D. (2016) The chicken or the egg? The direction of the relationship between mathematics anxiety and mathematics performance. *Frontiers in Psychology, 6,* 1987.

Close, S. (2006). The junior cycle curriculum and the PISA mathematics framework. *The Irish Journal of Education, 37*, 53-78

Cosgrove, J., Perkins, R., Shiel, G., Fish, R., & McGuinness, L. (2012). *Teaching and learning in Project Maths: Insights from teachers who participated in PISA 2012*. Dublin: Educational Research Centre.

Cosgrove, J., Shiel, G., Sofroniou, S., Zastrutzki, S., & Shortt, F. (2005). *Education for life: The achievements of 15-year-olds in Ireland in the second cycle of PISA*. Dublin: Educational Research Centre.

Department of Education and Skills. (2011). *Literacy and numeracy for learning and life: The national strategy to improve literacy and numeracy among children and young people, 2011-2020*. Dublin: Author.

Department of Education and Skills. (2012). *A framework for the Junior Cycle*. Dublin: Author.

Department of Education and Skills. (2015). *Framework for the Junior Cycle*. Dublin: Author.

Dweck, C. (2006). *Mindset: the new psychology of success. How we can learn to fulfil our potential.* New York: Ballantine Books.

Fox, S., & Surtees, L. (2010). *Mathematics across the curriculum: Problem-solving, reasoning and numeracy in primary schools*. London: Continuum Publishing.

Funke, J. (2010). Complex problem solving: A case for complex cognition? *Cognitive Processing*, Vol. 11, 133-142.

Jeffes, J., Jones, E., Cunningham, R., Dawson, A., Cooper, L., Straw, S., Sturman, L. and O'Kane, M. (2012). *Research into the impact of Project Maths on student achievement, learning and motivation: first interim report.* Slough: NFER.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R. and Dawson, A. (2013). *Research into the impact of Project Maths on student achievement, learning and motivation: final report.* Slough: NFER.

Lyons, M., Lynch, K., Close, S., Sheeran, E., & Boland, P. (2003). *Inside classrooms: a study of teaching and learning*. Dublin: Institute of Public Administration.

Merriman, B., Shiel, G., Cosgrove, J. and Perkins, R. (2014). *Project Maths and PISA 2012: Performance in Initial Project Maths schools and in non-Initial schools on PISA 2012 mathematics and problem solving and on Junior Certificate mathematics.* Dublin: Educational Research Centre.

Moran, G., Perkins, R., Cosgrove, J., & Shiel, G. (2013). *Mathematics in Transition Year: Insights of teachers from PISA 2012*. Dublin: Educational Research Centre.

National Council for Curriculum and Assessment/Department of Education and Skills. (2011a). *Junior Certificate mathematics syllabus: Foundation, ordinary, and higher level*. Dublin: NCCA/DES.

National Council for Curriculum and Assessment/Department of Education and Skills. (2011b). *Leaving Certificate mathematics syllabus: Foundation, ordinary, and higher level*. Dublin: NCCA/DES.

Organisation for Economic Cooperation and Development. (2013a). *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*. Paris: OECD publishing.

Organisation for Economic Cooperation and Development. (2013b). *What Students know and can do: Student Performance in Mathematics, Reading and Science (Volume I)*. Paris: OECD publishing.

Organisation for Economic Cooperation and Development. (2014). *Skills for Life: Student performance in problem solving (Volume V)*. Paris: OECD Publishing.

Schmidt, W.H., Burroughs, N.A., Zoido, P., & Houang, R. (2015). The role of schooling in perpetuating educational inequality: An international perspective. *Educational Researcher, 20(10),* 1-16.

Sobanski, J. (2002). *Visual math. See how math makes sense*. New York: Learning Express.

Tobias, S. (1993). *Overcoming math anxiety.* New York: W. W. Norton & Company.

Uttal, D., Meadow, N.G., Tipton, E., Hand, L., Aiden, A.R., Warren, C., & Newcombe, N.S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352-402.

Waj, J., Lubinski, D., & Benbow, C.J. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101(4)*, 817-835.

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